Topic 2: Linear Economic Models

Jacques Text Book (edition 3):

section 1.2 – Algebraic Solution of Simultaneous Linear Equations

section 1.3 – Demand and Supply Analysis

Content

- **Simultaneous Equations**
- **Market Equilibrium**
- **Market Equilibrium + Excise Tax**
- **Market Equilibrium + Income**

Solving Simultaneous Equations

Example

- **$4x + 3y = 11$** **(eq.1)**
- **$2x + \quad y = 5$** **(eq.2)**

Express both equations in terms of the same value of x (or y)

- **$4x = 11 - 3y$** **(eq.1')**
- **$4x = 10 - 2y$** **(eq.2')**

Hence

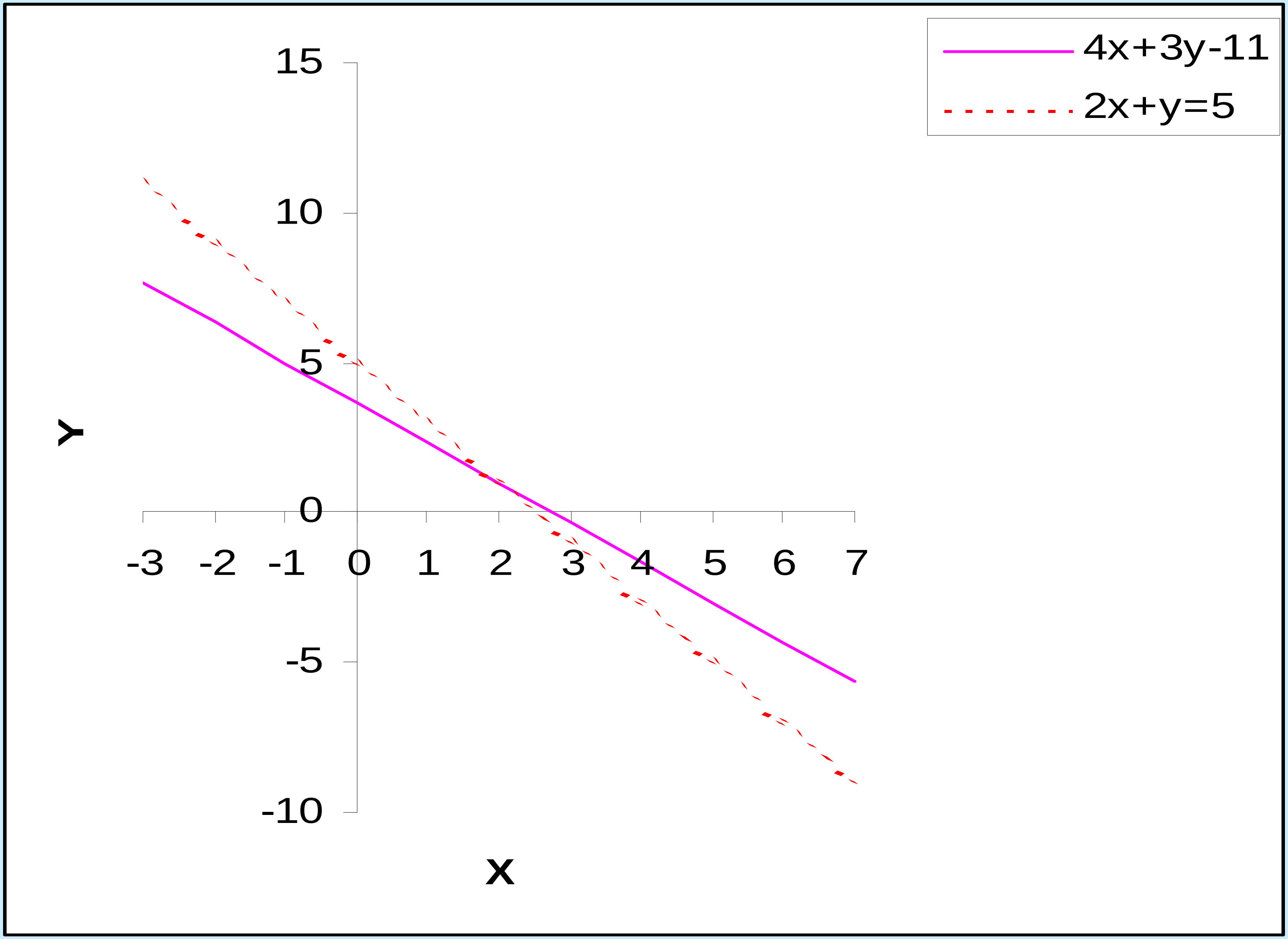
- **$11 - 3y = 10 - 2y$**

Collect terms

- **$11 – 10 = -2y + 3y$**
- **$y = 1$**

Compute x

- **$4x = 10 - 2y$**
- **$4x = 10 – 2 = 8$**
- **$x = 2$**


4x+3y-11
2x+y=5
15
10
5
0
-5
-10
-3
-2
-1
0
1
2
3
4
5
6
7
Y
X

Note that if the two functions do not intersect, then cannot solve equations simultaneously…..

- **$x - 2y = 1$ (eq.1)**
- **$2x - 4y = -3$ (eq.2)**

Step 1

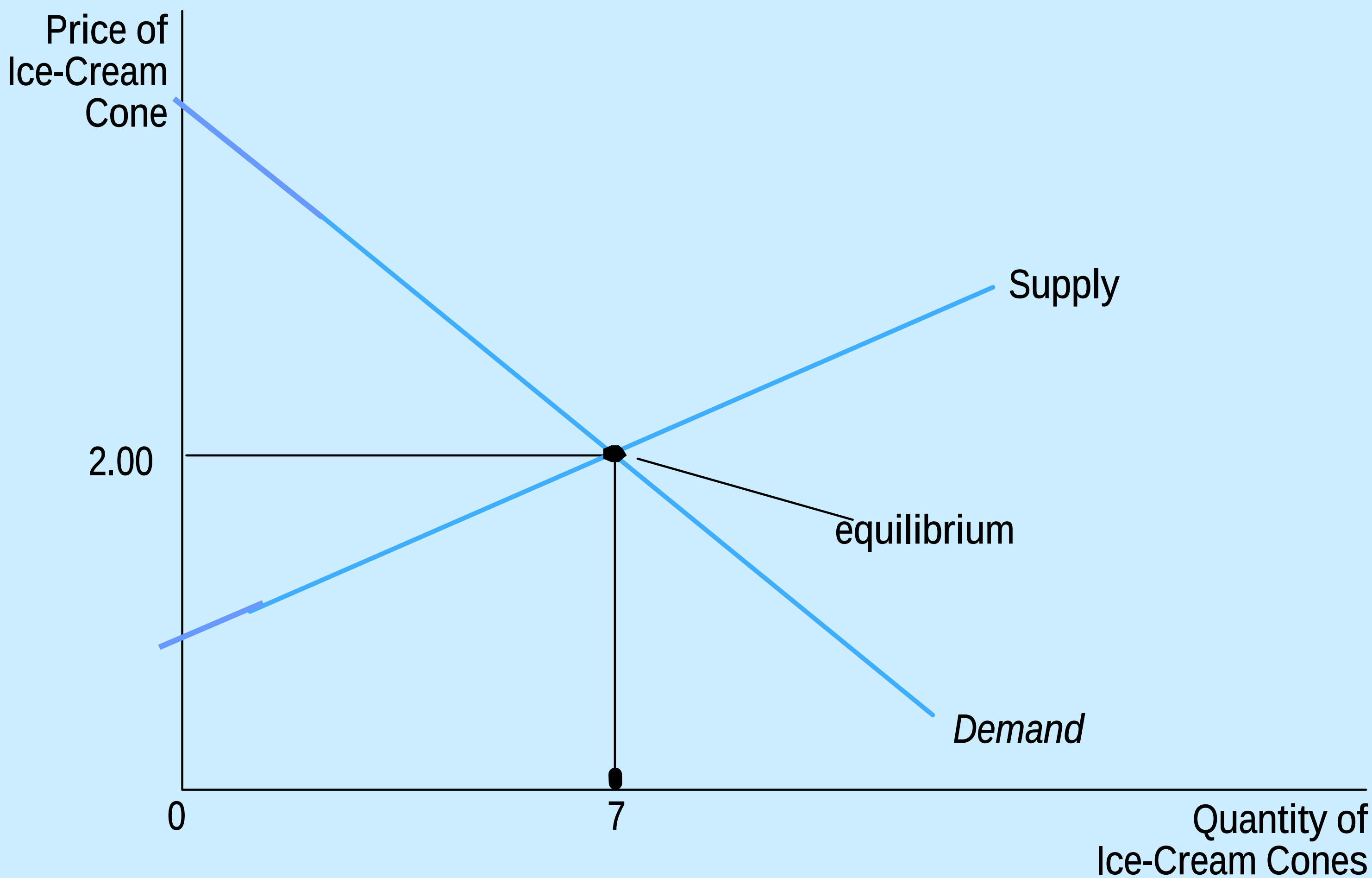
- **$2x = 2 + 4y$ (eq.1*)**
- **$2x = -3 + 4y$ (eq.2*)**

Step 2

- **$2 + 4y = -3 + 4y$ BUT => $2+3 = 0$…………**
- No Solution to the System of Equations

Solving Linear Economic Models

- ***Quantity Supplied***: amount of a good that sellers are willing and able to sell
- ***Supply curve:*** upward sloping line relating price to quantity supplied
- ***Quantity Demanded***: amount of a good that buyers are willing and able to buy
- ***Demand curve:*** downward sloping line relating price to quantity demanded
- **Market Equilibrium:** quantity demand = quantity supply


Price of
Ice-Cream
Cone
2.00
Supply
equilibrium
Demand
0
7
Quantity of
Ice-Cream Cones

Finding the equilibrium price and quantity levels…..

- ***<u>In general,</u>***

Demand: $Q_D = a + bP$ (with $b<0$)

Supply: $Q_S = c + dP$ (with $d>0$)

- Set $Q_D = Q_S$ and solve simultaneously for

$P^e = (a - c)/(d - b)$

- Knowing P^e, find Q^e given the demand/supply functions

$Q^e = (ad - bc)/(d - b)$

Example 1

Demand	$Q_D = 50 - P$	(i)
Supply	$Q_S = -10 + 2P$	(ii)

Set $Q_D = Q_S$ find market equilibrium P and Q

- $50 - P = -10 + 2P$
- $3P = 60$
- **$P = 20$**

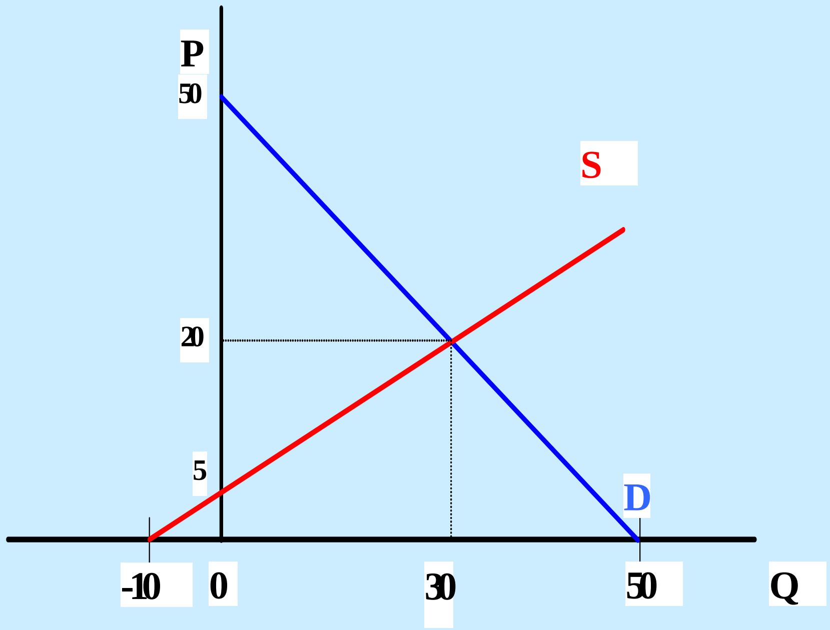
Knowing P, find Q

- $Q = 50 - P$
- $= 50 - 20 = 30$

Check the solution

- i) $30 = 50 - 20$ and (ii) $30 = -10 + 40$

In both equations if P=20 then Q=30



Example 2

Demand	$Q_D = 84 - 3P$	(i)
Supply	$Q_S = -60 + 6P$	(ii)

Set $Q_D = Q_S$ to find market equilibrium

- $84 - 3P = -60 + 6P$
- $144 = 9P$
- $P = 16$

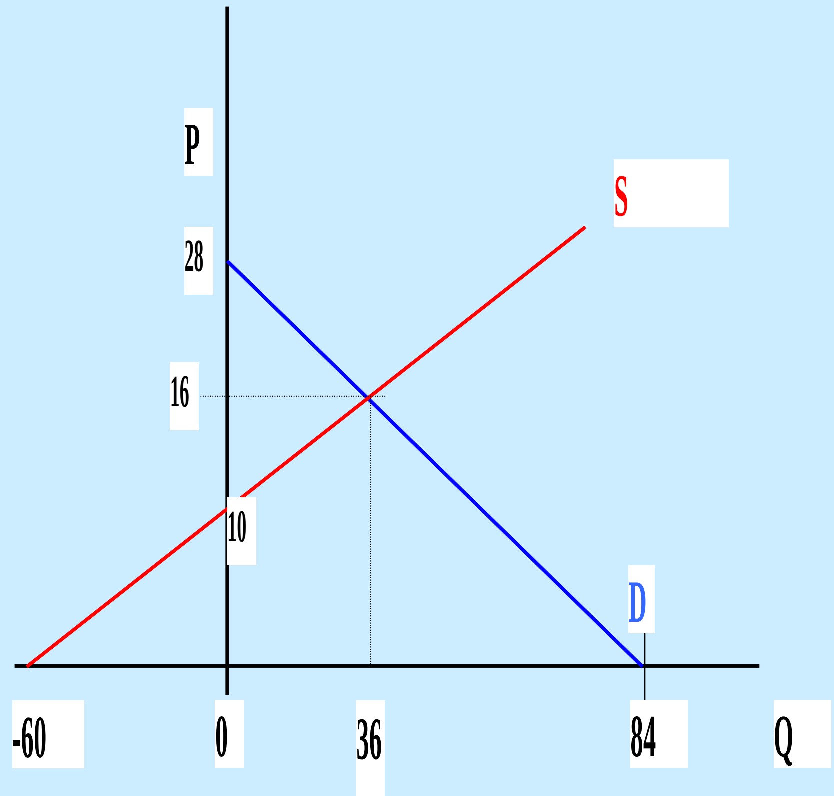
Knowing P, find Q

- $Q = -60 + 6P$
- $= -60 + 96 = 36$

Check the solution

- $36 = 84 - (3*16)$ and
- $36 = -60 + (6*16)$

In both equations if P=16 then Q=36



Market Equilibrium + Excise Tax

- Impose a tax t on suppliers per unit sold……
- Shifts the supply curve to the left
- Q_D = a – bP
- Q_S = d + eP *with no tax*
- Q_S = d + e(P – t) *with tax t on suppliers*

So from example 1….

- Q_D = 50 – P,
- Q_S = – 10 + 2P becomes
- Q_S = – 10 + 2(P-t) = – 10 + 2P – 2t

cont…..

Continued…..

Write Equilibrium P and Q as functions of t

- **Set $Q_D = Q_S$**
- $50 – P = – 10 + 2P – 2t$
- $60 = 3P - 2t$
- $3P = 60 + 2t$
- $P = 20 + {}^{2}/_{3}t$

Knowing P, find Q

- $Q = 50 – P$

Comparative Statics: effect on P and Q of ↑t

(i) As ↑ t, then ↑ P paid by consumers by $^2/_3$t

⇒ remaining tax ($^1/_3$) is paid by suppliers

total tax t = $^2/_3$t + $^1/_3$t

Consumers pay ***Suppliers pay***

Price consumers pay – price suppliers receive = total tax t

(ii) and ↓ Q by $^2/_3$t *, reflecting a shift to the left of the supply curve*

For Example let t = 3

- **Q_D = 50 – P**
- **Q_S =** – 10 + 2(P-t)

  = **– 16 + 2P**

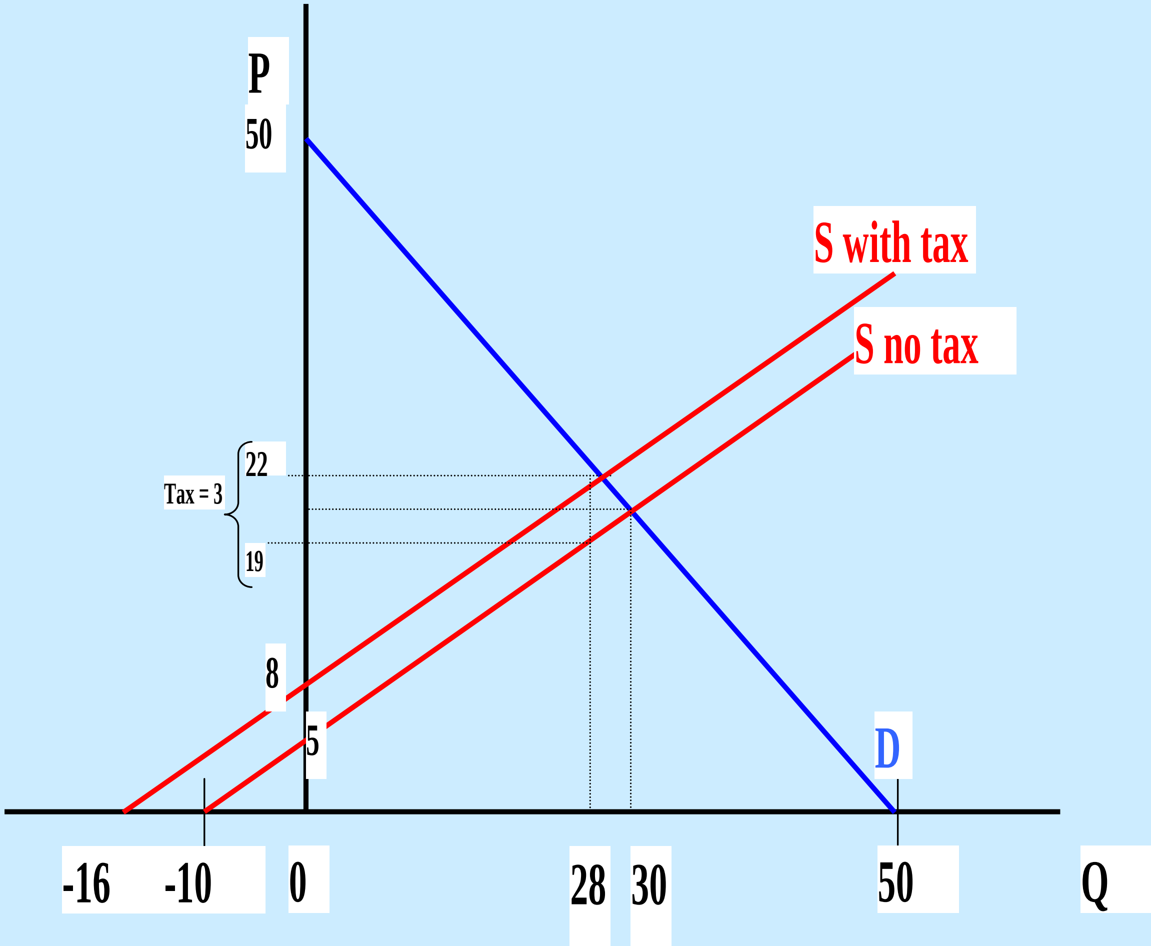
- **New equilibrium Q = 28**

  (Q = 30 - $^2/_3$t)
- **New equilibrium P = 22**

  (P = 20 + $^2/_3$t)

Supplier Price = 19

Tax Revenue = P*Q = 3*28 = 84



Another Tax Problem….

$$Q_D = 132 - 8P$$
$$Q_S = -6 + 4P$$

- **Find the equilibrium P and Q.**
- **How does a per unit tax t affect outcomes?**
- **What is the equilibrium P and Q if unit tax t = 4.5?**

Solution…..

(i) Market Equilibrium values of P and Q

- **Set $Q_D = Q_S$**

  $132 – 8P = – 6 + 4P$

  $12P = 138$

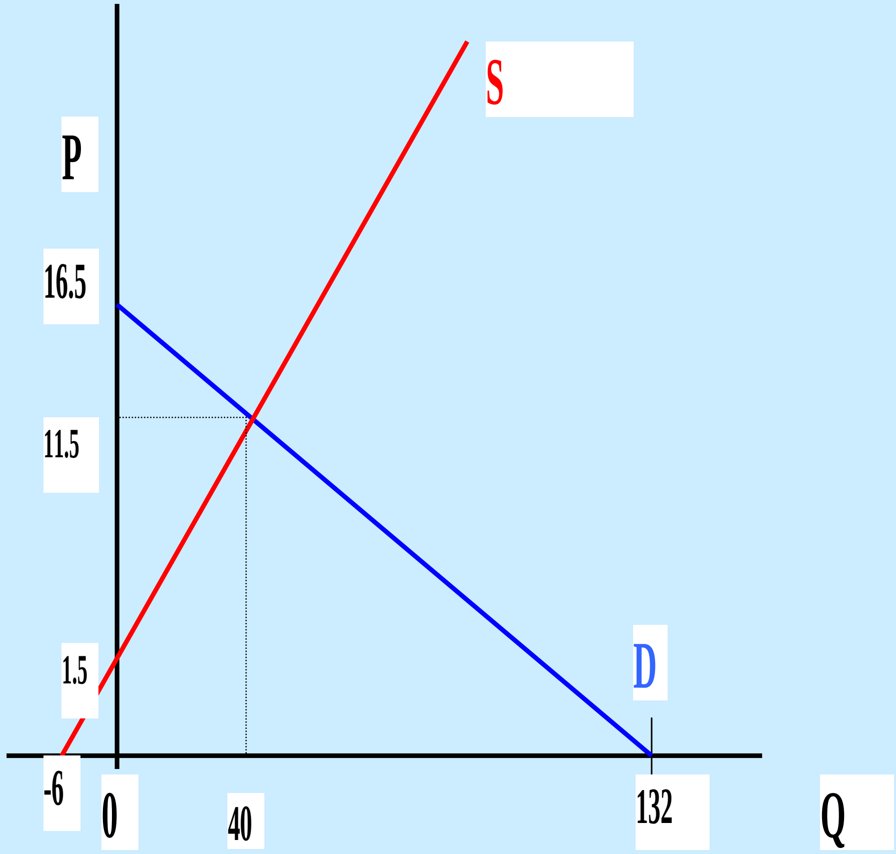
  $P = 11.5$

- **Knowing P, find Q**

  $Q = – 6 + 4P$

- $= – 6 + 4(11.5) =$ **40**

- **Equilibrium values: P = 11.5 and Q= 40**



(ii) The Comparative Statics of adding a tax……

$$Q_D = 132 - 8P$$
$$Q_S = -6 + 4(P - t) = -6 + 4P - 4t$$

Set $Q_D = Q_S$

$132 - 8P = -6 + 4P - 4t$

$12P = 138 + 4t$

$P = 11.5 + {}^{1}/_{3}\, t = 13$ if $t = 4.5$

Imposing t => ↑consumer P by ${}^{1}/_{3}t$, supplier pays ${}^{2}/_{3}t$

Knowing P, find Q

$$Q = 132 - 8(13) = 28$$

(iii) If per unit t = 4.5

Tax = 0

Consumer Price = 11.5

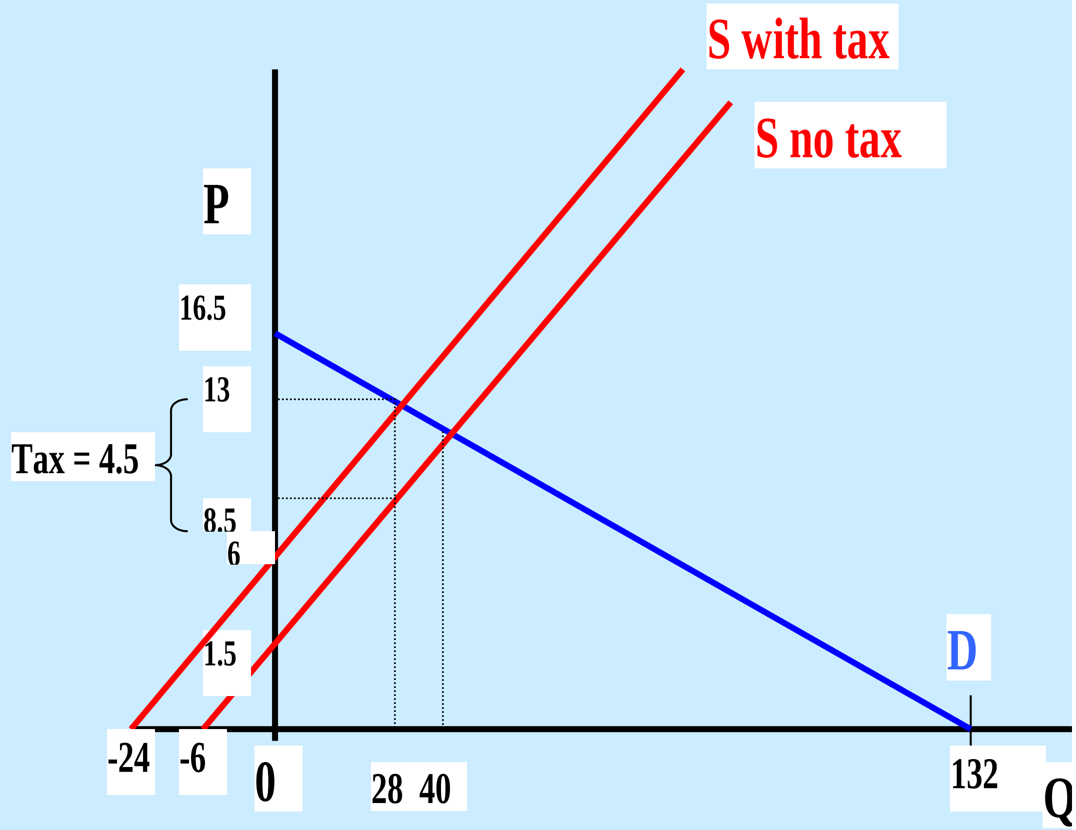
Supplier Price = 11.5

Tax = 4.5

Consumer Price = 13

Supplier Price = 8.5

Tax Revenue = P*Q = 4.5*28 =126



Market Equilibrium and Income

- Let $Q_D = a + bP + cY$

Example: the following facts were observed for a good,

- Demand = 110 when P = 50 and Y = 20
- When Y increased to 30, at P = 50 the demand = 115
- When P increased to 60, at Y = 30 the demand = 95

(i) Find the Linear Demand Function Q_D?

Rewriting the facts into equations:

- $110 = a + 50b + 20c$ eq.1
- $115 = a + 50b + 30c$ eq.2
- $95 = a + 60b + 30c$ eq.3

To find the demand function

$Q_D = a + bP + cY$

we need to solve these three equations simultaneously for a, b, and c

Rewriting 1 and 2

- a = 110 - 50b - 20c (eq.1*)
- a = 115 - 50b - 30c (eq.2*)
- =>
- 110 – 50b – 20c = 115 – 50b – 30c
- 10c = 5
- **c = ½**

Rewriting 1 & 3 given c = ½

- 110 = a + 50b + 10 (eq.1*)
- 95 = a + 60b + 15 (eq.3*)
- a = 100 – 50b (eq.1*)
- a = 80 – 60b (eq.3*)
- 100 – 50b = 80 – 60b
- 10b = -20
- **b = -2**

Given b= -2 and c = ½ , solve for a

- a = 110 – 50b – 20c
  eq.1’
- a = 110 + 100 – 10 = 200

- $\Rightarrow$ **Q_D = 200 -2P + ½Y**

Now, let $Q_S = 3P – 100$

Describe fully the comparative static's of the model using Q_D and Q_S equations?

Set $Q_D = Q_S$ for equilibrium values of P and Q

- $200 - 2P + ½Y = 3P – 100$
- $5P = 300 + ½Y$
- $P = 60 + {}^{1}/_{10}Y$

Knowing P, find Q

- $Q = 3(60 + {}^{1}/_{10}Y) - 100$
- $= 80 + {}^{3}/_{10}Y$

What is equilibrium P and Q when Y = 20

- $P = 60 + {}^{1}/_{10}Y$
- $P = 60 + {}^{1}/_{10}(20) = 62$

i.e $\uparrow$ P by ${}^{1}/_{10}$ of 20 = 2

- $Q = 80 + {}^{3}/_{10}Y$
- $Q = 80 + {}^{3}/_{10}(20) = 86$

i.e $\uparrow$ Q by ${}^{3}/_{10}$ of 20 = 6

Qd = 200 – 2P + ½ Y
Qs = 3P – 100

- Finding Intercepts:

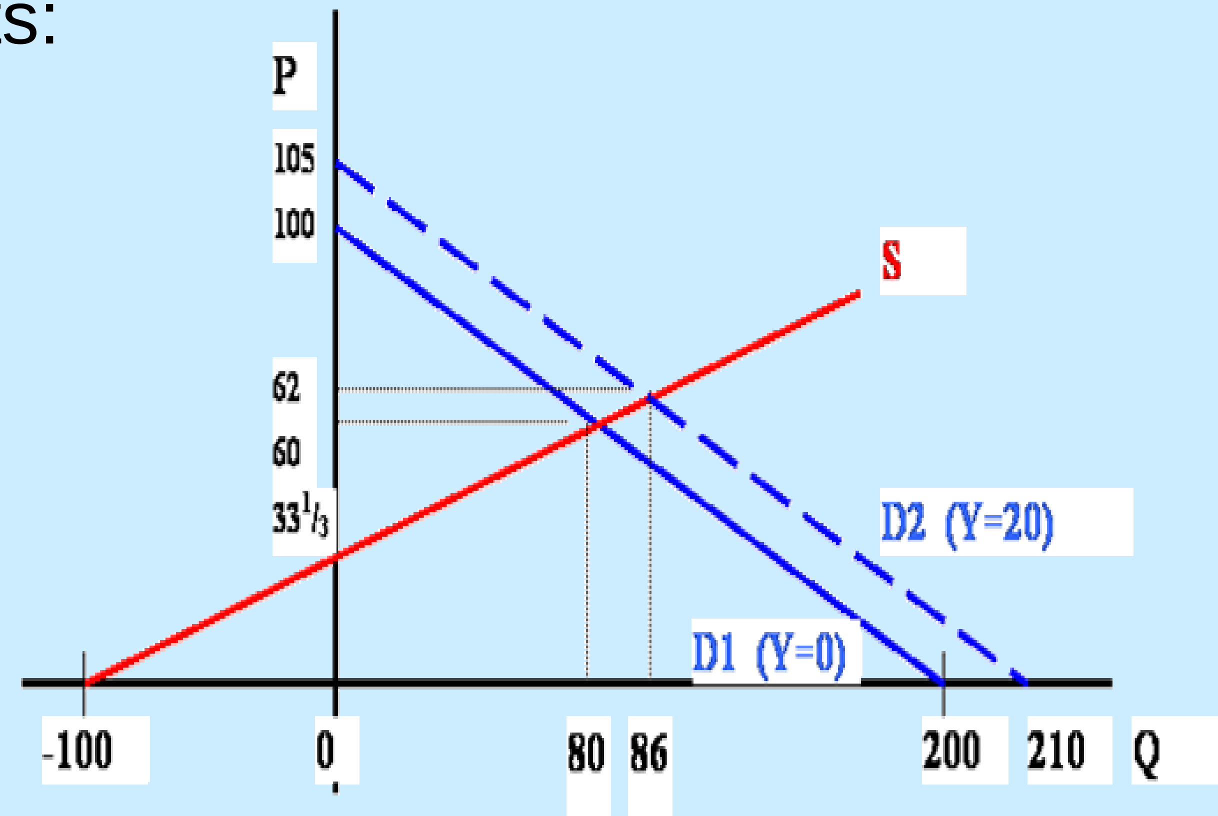
S(Q,P): (-100, 0)
and (0, $33^1/_3$)

Y=0:

D1(Q,P): (200, 0)
and (0, 100)

Y=20:

D2(Q,P): (210, 0)
and (0, 105)



Questions Covered
Topic 2: Linear Economic Models

- Algebraic Solution of Simultaneous Linear Equations
- Solving for equilibrium values of P and Q
- Impact of tax on equilibrium values of P and Q
- Impact of Income on Demand Functions and on equilibrium values of P and Q