Topic 2: Linear Economic Models

(i) Market Equilibrium
(ii) Market Equilibrium + Excise Tax

Lecture Notes:
sections 2.1 and 2.2

Jacques Text Book (edition 2):
section 1.2 – Algebraic Solution of Simultaneous Linear Equations

section 1.3 – Demand and Supply Analysis

ASIDE:
Solving Simultaneous Equations

⇨ Plot on a graph and then solve to find common co-ordinates……

OR

⇨ Solve Algebraically

Eg.

$4x + 3y = 11$ eq.1

$2x + y = 5$ eq.2

1. Express both eq. in terms of the same value of x (or y)

$4x = 11 - 3y$ eq.1

$4x = 10 - 2y$ eq.2’

2. Substitute value of eq.1 into eq.2'

   ⇨ **11 - 3y = 10 - 2y**

3. collect terms

   ⇨ **11 – 10 = -2y + 3y**

   ⇨ **1 = y**

4. now compute x

   ⇨ **4x = 10 - 2y**

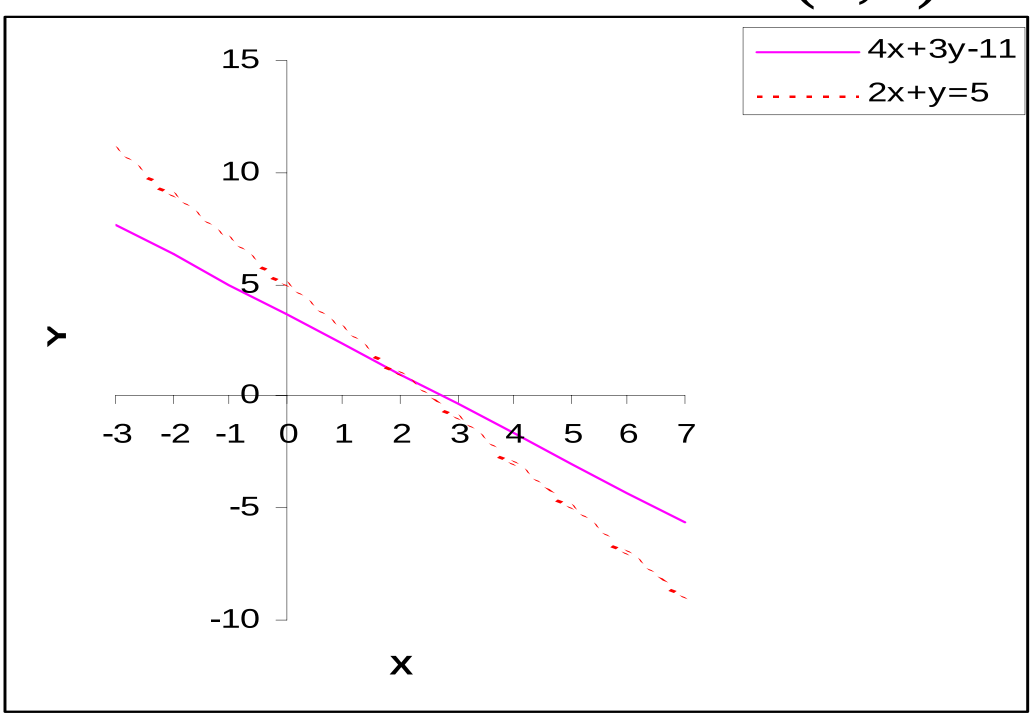
   ⇨ **4x = 10 – 2 = 8**

   ⇨ **x = 2**

5. check the solution

⇨ in both equations, when x = 2, y = 1

⇨ the two lines intersect at (2,1)



Note that if the two functions do not intersect, then cannot solve equations simultaneously…..

$x - 2y = 1$ **eq.1**

$2x - 4y = -3$ **eq.2**

Step 1

$2x = 2 + 4y$ **eq.1'**

$2x = -3 + 4y$ **eq.2**

Step 2

$2 + 4y = -3 + 4y$ **BUT =>**

$2+3 = 0$…………

No Solution to the System of Equations

Solving Linear Economic Models

Market Equilibrium

Quantity Demanded = Quantity Supplied

Finding the equilibrium price and quantity levels…..

<u>In general,</u>

Demand Function: $Q_D = a + bP$
Supply Function: $Q_S = c + dP$

- Set $Q_D = Q_S$ and solve simultaneously for $P^e = (a - c)/(d - b)$
- Knowing P^e, find Q^e given the demand/supply functions
- $Q^e = (ad - bc)/(d - b)$

Eg.1

$$Q_D = 50 - P \qquad \text{(i)}$$
$$Q_S = 20 + 2P \qquad \text{(ii)}$$

⇨ **Set $Q_D = Q_S$**

$$50 - P = 20 + 2P$$
$$3P = 30$$
$$\mathbf{P = 10}$$

⇨ **Knowing P, find Q**

$$Q = 50 - P$$
$$= 50 - 10 = 40$$

⇨ **Check the solution**

i) $40 = 50 - 10$ and (ii) $40 = 20 + (2*10)$

In both equations if P=10 then Q=40

Changes in Demand or Supply…

Shift the curves and results in a new equilibrium price and quantity

Section 2.2 Notes: Market Equilibrium + Excise Tax

Impose a tax t on suppliers per unit sold……
Shifts the supply curve to the left

$Q_D = a + bP$
$Q_S = d + eP$ *with no tax*
$Q_S = d + e(P-t)$ *with tax t on suppliers*

$Q_D = 50 – P$, and $Q_S = 20 + 2P$ becomes
$Q_S = 20 + 2(P-t)$

Write Equilibrium P and Q as functions of t

⇨ **Set $Q_D = Q_S$**

$$50 - P = 20 + 2(P-t)$$
$$30 = 3P - 2t$$
$$3P = 30 + 2t$$
$$P = 10 + {}^{2}/_{3}t$$

Consumer Price → $P = 10 + {}^{2}/_{3}t$

⇨ **Knowing P, find Q**

$$Q = 50 - P$$
$$= 50 - (10 + {}^{2}/_{3}t)$$
$$= 40 - {}^{2}/_{3}t$$

Comparative Statics: effect on P and Q of ↑t

(i) As ↑ t, then ↑ P paid by consumers by $^{2}/_{3}t$

⇒ remaining tax ($^{1}/_{3}$) is paid by suppliers

$$\text{total tax } t = {}^{2}/_{3}t + {}^{1}/_{3}t$$

Consumers pay ($^{2}/_{3}t$) — Suppliers pay ($^{1}/_{3}t$)

Price consumers pay – price suppliers receive = total tax t

e.g. t = £3
Consumer P: £12 (pre-tax eq. p + $^{2}/_{3}t$)
Supplier P: £9 (pre-tax eq. p – $^{1}/_{3}t$)

(ii) and ↓ Q by $^{2}/_{3}t$, reflecting a shift to the left of the supply curve

Another Tax Problem….

$$Q_D = 132 - 8P$$
$$Q_S = 6 + 4P$$

- **(i) Find the equilibrium P and Q.**
- **(ii) How does a per unit tax t affect outcomes?**
- **(iii) What is the equilibrium P and Q if unit tax t = 4.5?**

Solution…..

(i) Equilibrium values

⇨ **Set $Q_D = Q_S$**

$$132 - 8P = 6 + 4P$$

$$12P = 126$$

$$P = 10.5$$

⇨ **Knowing P, find Q**

$$Q = 6 + 4P$$

$$= 6 + 4(10.5) = \mathbf{48}$$

Equilibrium values: P = 10.5 and Q=48

(ii) The comparative Statics of adding a tax……

$$Q_D = 132 - 8P$$
$$Q_S = 6 + 4(P - t)$$

⇨ **Set $Q_D = Q_S$**

$$132 - 8P = 6 + 4(P - t)$$
$$12P = 126 + 4t$$
$$P = 10.5 + {}^{1}/_{3}\, t$$

⇨ **Knowing P, find Q**

$$Q = 6 + 4[P - t]$$
$$= 6 + 4[(10.5 + {}^{1}/_{3}\, t) - t]$$
$$= 48 - {}^{8}/_{3}\, t$$

Imposing t => ↑ consumer P by ${}^{1}/_{3}t$, *supplier pays* ${}^{2}/_{3}t$, *and* ↓ *Q by* ${}^{8}/_{3}\, t$

(iii) If per unit t = 4.5

$$P = 10.5 + {}^{1}/_{3}\ (4.5) = 12$$

Consumer P: £12 (pre-tax eq. p + ${}^{1}/_{3}t$*)*
Supplier P: £7.5 (pre-tax eq. p – ${}^{2}/_{3}t$*)*

$$Q = 48 - {}^{8}/_{3}\ (4.5) = 36$$

Market Equilibrium and Income

Increase in Income Y => Shift Out of Demand Curve => $\uparrow Q_D$ and $\uparrow P$

$$Q_D = a + bP + cY$$
$$Q_S = d + eP$$

Let,

$$\mathbf{Q_D = 200 - 2P + \tfrac{1}{2}Y}$$
$$\mathbf{Q_S = 3P - 100}$$

Given the above Demand and Supply functions, what is the impact on the Market Equilibrium of Y increasing from 0 to 20?

⇨ **Set $Q_D = Q_S$**

$$200 - 2P + \frac{1}{2}Y = 3P - 100$$

$$5P = 300 + \frac{1}{2}Y$$

$$P = 60 + {}^{1}/_{10}Y$$

⇨ **Knowing P, find Q**

$$Q = 3(60 + {}^{1}/_{10}Y) - 100$$

$$= 80 + {}^{3}/_{10}Y$$

As ↑ Y => ↑ P by ${}^{1}/_{10}$ of ↑Y, and ↑ Q by ${}^{3}/_{10}$ of ↑Y

What is equilibrium P and Q when Y = 20

$P = 60 + {}^{1}/_{10}Y$

$P = 60 + {}^{1}/_{10}(20) = 62$

i.e ↑P by ${}^{1}/_{10}$ of 20 = 2

$Q = 80 + {}^{3}/_{10}Y$

$Q = 80 + {}^{3}/_{10}(20) = 86$

i.e ↑P by ${}^{3}/_{10}$ of 20 = 6

Qd = 200 – 2P + ½ Y

Qs = 3P – 100

Finding Intercepts:

S (Q, P): (-100, 0) and (0, $33^1/_3$)

Y=0:

D1 (Q,P): (200, 0) and (0, 100)

Y=20:

D2 (Q,P): (210, 0) and (0, 105)