MSc Econometrics II

Problem set 1

To hand in by January 27th.

Exercise 1

1. Consider the following sample autocorrelation function, extracted from a sample of 200 quarterly observations.

k	1	2	3	4	5	6	7	8	9	10
$\widehat{\rho}$	0.83	0.71	0.60	0.45	0.44	0.35	0.29	0.20	0.11	-0.01

Please explain the meaning of sample autocorrelation function. Does the above pattern indicate that an autoregressive or moving average representation is more appropriate? Why?

2. Now consider the sample partial autocorrelation function.

k	1	2	3	4	5	6	7	8	9	10
$\widehat{\theta}_{kk}$	0.83	0.16	-0.09	0.05	0.04	-0.05	0.01	.010	-0.03	0.01

Please explain the meaning of sample partial autocorrelation function. Why is it the first partial autocorrelation equal to the first autocorrelation coefficient (0.83)?

Does the above pattern indicate that an autoregressive or moving average process representation is more appropriate? Why?

Exercise 2

1. Consider a first-order autoregressive process, $AR(1)$

$$y_t = \vartheta y_{t-1} + \varepsilon_t$$

where $|\vartheta| < 1$ and $\{\varepsilon_t\}$ is a white noise stochastic process, $\varepsilon_t \sim IID(0, \sigma^2)$.

Show that the $AR(1)$ process can be represented as a moving average process of infinite order, $MA(\infty)$

2. Consider a first-order moving average process, $MA(1)$

$$y_t = \varepsilon_t + \phi \varepsilon_{t-1}$$

where $|\phi| < 1$ and $\{\varepsilon_t\}$ is a white noise stochastic process, $\varepsilon_t \sim IID(0, \sigma^2)$.

Show that the $MA(1)$ process can be represented as an autoregressive process of infinite order, $AR(\infty)$

Exercise 3

If y_t is a stationary $AR(1)$ process

$$y_t = \phi y_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim IID(0, \sigma^2)$, with $t = 1, 2, ..., T$

1. Show that
$$Cov(y_t, y_{t-j}) = Cov(y_t, y_{t+j}) = \frac{\phi^j \sigma^2}{1 - \phi^2}$$

2. Use the result above to show that the correlation between y_t and y_{t-j} is ϕ^j

2. Consider a stationary $AR(2)$ process

$$Y_t = \delta + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$$

Calculate the autocovariances $cov(Y_t, Y_{t-1})$, $cov(Y_t, Y_{t-2})$, $cov(Y_t, Y_{t-3})$.

Exercise 4

Show that the $AR(1)$ process with $\phi = 1$ is not covariance stationary.

$$Y_t = \alpha + Y_{t-1} + \varepsilon_t$$

Exercise 5

Please state whether the following statements are true or false. Explain.

1. Not all moving-average processes with finite coefficients are stationary
2. An $ARMA(p, q)$ process is stationary iff the AR part of the model is stationary.

Exercise 6

What is the condition for stationarity for an $AR(2)$ process? Can you generalize this condition for an $AR(p)$ process?

Exercise 7 (Rank condition as no multicollinearity at the limit)

Consider a stationary and ergodic $AR(1)$ process

$$y_t = \delta + \phi y_{t-1} + \varepsilon_t$$

where $\{\varepsilon_t\}$ is a white noise stochastic process, $\varepsilon_t \sim IID(0, \sigma^2)$,and $t = 1, 2, ..., T$. Write the $AR(1)$ process as:

$$y_t = \mathbf{x}_t' \boldsymbol{\beta} + \varepsilon_t$$

where $\mathbf{x}_t = (1, y_{t-1})'$ and $\boldsymbol{\beta} = (\delta, \phi)'$

Show that for T sufficiently large, the sample cross moment of the regressors $\mathbf{S}_{xx} = \frac{1}{T}\mathbf{X}'\mathbf{X} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_t\mathbf{x}_t'$ is nonsingular. [Hint: check that the moment matrix $E(\mathbf{x}_t\mathbf{x}_t')$ is nonsingular (and hence finite). Denote this matrix by $\mathbf{\Sigma}_{xx}$].