Moral Hazard and Joint Liability

Output Y can take 2 values:

$$Y = \begin{cases} Y^H & \text{with probability } p \\ 0 & \text{with probability } (1-p) \end{cases}$$

where p is the probability of success or effort.

Effort is costly. The effort cost function is:

$$C(p) = \frac{1}{2}\gamma p^2$$

where $\gamma > 0$

1. **First best**: social surplus maximization; no asymmetric information.

$$\max_p \left[pY^H - \frac{1}{2}\gamma p^2 \right]$$

Social surplus= private profit of the lender + private profit of the borrower

First Order Condition

Take the first derivative with respect to p and set it equal to zero.

$$Y^H - \gamma p = 0$$

Solve for p:

$$p^*_{FB} = \frac{Y^H}{\gamma}$$

where $Y^H < \gamma$ so that we have an interior solution.

Zero-profit condition

What is the loan repayment, r? In order to calculate the repayment, we have to consider the zero-profit condition for the lender.

$$\begin{aligned} \rho &= pr + (1-p)\,0 \\ \rho &= pr \end{aligned}$$

where ρ is the opportunity cost of capital. Solve for r

$$r^*_{FB} = \frac{\rho}{p^*_{FB}}$$

$$r^*_{FB} = \frac{\rho}{\frac{Y^H}{\gamma}} = \frac{\gamma\rho}{Y^H}$$

2. **Second best**: Ex-post asymmetric information; Individual contracts.

We have to maximize the expected net return for the borrower

$$\max \left[p \left(Y^H - r \right) + (1-p)\, 0 - \frac{1}{2}\gamma p^2 \right]$$
$$\max \left[p \left(Y^H - r \right) - \frac{1}{2}\gamma p^2 \right]$$

First Order Condition

Take the first derivative with respect to p and set it equal to zero.

$$Y^H - r - \gamma p = 0$$

$$p = \frac{Y^H - r}{\gamma}$$

Note that the effort in the second best is less than the effort in the first best case $\left(p^*_{FB} = \frac{Y^H}{\gamma} \right)$.

Zero-profit condition

What is the loan repayment, r? In order to calculate the repayment, we have to consider the zero-profit condition for the lender.

$$\begin{aligned} \rho &= pr + (1-p)\, 0 \\ \rho &= pr \end{aligned}$$

The lender will choose r, given the level of effort p chosen by the borrower

$$\rho = \frac{Y^H - r}{\gamma} r$$

$$\gamma \rho = r Y^H - r^2$$

$$r^2 - r Y^H + \rho \gamma = 0$$

$$r = \frac{Y^H \pm \sqrt{\left(Y^H\right)^2 - 4\rho\gamma}}{2}$$

Now substitute the values of r into the First order condition

$$p = \frac{Y^H - r}{\gamma}$$

$$p = \frac{Y^H}{\gamma} - \frac{r}{\gamma}$$

$$p = \frac{Y^H}{\gamma} - \frac{Y^H \pm \sqrt{\left(Y^H\right)^2 - 4\rho\gamma}}{2\gamma}$$

$$p = \frac{Y^H \pm \sqrt{\left(Y^H\right)^2 - 4\rho\gamma}}{2\gamma}$$

The borrower chooses the equilibrium value with higher p. Why? The bank is indifferent and the borrower is better off.

$$p^*_{IND} = \frac{Y^H + \sqrt{\left(Y^H\right)^2 - 4\rho\gamma}}{2\gamma}$$

3. **Joint liability**. The borrower pays the joint liability payment c if the partner is unsuccessful and defaults.

We have to maximize the expected net return for the borrower

$$\max_p \left[p\left(Y^H - r\right) - \frac{1}{2}\gamma p^2 - p\left(1-p\right)c \right]$$

Note that we assume that both partners will choose the effort cooperatively

$$\max_p \left[pY^H - pr - \frac{1}{2}\gamma p^2 - pc + p^2 c \right]$$

First Order Condition

$$Y^H - r - \gamma p - c + 2pc = 0$$

$$p\left(\gamma - 2c\right) = Y^H - r - c$$

$$p = \frac{Y^H - r - c}{\gamma - 2c}$$

Zero-profit condition
The bank's zero profit condition is

$$pr + p\left(1-p\right)c = \rho$$

solve for r:

$$r = \frac{\rho - cp\left(1-p\right)}{p}$$

Now substitute into the First Order Condition:

$$p = \frac{Y^H - r - c}{\gamma - 2c}$$

$$p\left(\gamma - 2c\right) = Y^H - \frac{\rho - cp\left(1-p\right)}{p} - c$$

$$p^2\left(\gamma - 2c\right) = pY^H - \rho + cp\left(1-p\right) - cp$$

$$p^2(\gamma - 2c) = pY^H - \rho - cp^2$$

$$p^2(\gamma - c) - pY^H + \rho = 0$$

$$p = \frac{Y^H \pm \sqrt{(Y^H)^2 - 4\rho(\gamma - c)}}{2(\gamma - c)}$$

As before, the borrower chooses the higher root.

$$p_{JL}^* = \frac{Y^H + \sqrt{(Y^H)^2 - 4\rho(\gamma - c)}}{2(\gamma - c)}$$

Now, let's compare p_{JL}^* and p_{IND}^*

$$p_{IND}^* = \frac{Y^H + \sqrt{(Y^H)^2 - 4\rho(\gamma - c)}}{2\gamma}$$

Remember that we assumed that $Y^H < \gamma$. Then it must be that $c < \gamma$. Why? Borrowers cannot pay more than what the project yields ($c < Y^H$) If $c < \gamma$, then it must be that:

- Numerator is higher for p_{JL}^*
- Denominator is lower for p_{JL}^*

$\Rightarrow p_{JL}^* > p_{IND}^*$

This means that the equilibrium value of p (effort, probability of success, measure of repayment) is higher under joint liability. Joint liability reduces moral hazard. through peer monitoring

Remember that we have made two (implicit) assumptions:

- Coordination of effort
- Perfect monitoring is costless