Self Assessment Solutions

Linear Economic Models

1. Demand and supply in a market are described by the equations

Qd = 66-3P
Qs = -4+2P

(i) Solve algebraically to find equilibrium P and Q

In equilibrium Qd = Qs
66-3P = -4+2P
-3P-2P = -4-66
-5P = -70
5P = 70
P* = 14

Qd = Qs = 66-3P = 66-3(14) = 66-42 = 24 = Q*

(ii) How would a per unit sales tax t affect this equilibrium and comment on how the tax is shared between producers and consumers

Sales tax reduces suppliers price by t (P-t)
Supply curve becomes: Qs = -4+2(P-t)
In equilibrium Qd = Qs
66-3P = -4+2(P-t)
66-3P = -4+2P-2t
-3P-2P = -4-2t-66
-5P = -70-2t
5P = 70+2t
P = 14+$^{2}/_{5}$t

Qd = Qs = 66-3P = 66-3(14+$^{2}/_{5}$t) = 66-42-$^{6}/_{5}$t = 24-$^{6}/_{5}$t

Equilibrium price increases by $^{2}/_{5}$ of the tax. This implies that the supplier absorbs $^{3}/_{5}$ of the tax and receives a price P-$^{3}/_{5}$t for its goods. The consumer pays $^{2}/_{5}$ of the tax. Equilibrium quantity falls by $^{6}/_{5}$t.

(iii) What is the equilibrium P and Q if the per unit tax is t=5

t = 5, Qs = -4+2(P-5) = -4+2P-10 = -14+2P
In equilibrium Qd = Qs
66-3P = -14+2P
-5P = -14-66
-5P = -80
5P = 80
P = 16 (i.e. 14+$^{2}/_{5}$t)

Qd = Qs = 66-3P = 66-3(16) = 18 (i.e. 24-$^{6}/_{5}$t)

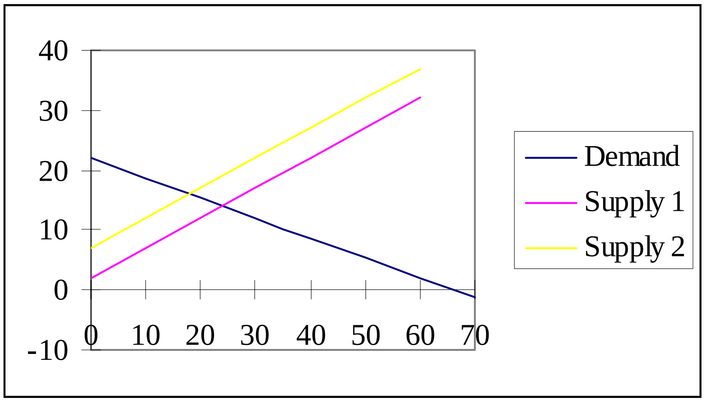
(iv) Illustrate the pre-tax equilibrium and the post-tax equilibrium on a graph

Qd = 66-3P   Qs = -4+2P

Let P = 0
Qd = 66
P = 22-Qd/3 (Inverse Demand)
Let Qd = 0
P = 22

Let P = 22
Qs = -4+2(22) = -4+44 = 40
P = 2+Qs/2 (Inverse Supply)
Let Qs = 0
P = 2

Qs = -14+2P
Let P = 22
Qs = -14+2(22) = -14+44 = 30
P = 7+Qs/2
Let Qs = 0
P = 7



Fill in equilibrium before tax, equilibrium after tax, amount paid by consumer, amount paid by producer.

2. The demand and supply functions of a good are given by

Qd = 110-5P

Qs = 6P

where P, Qd and Qs denote price, quantity demanded and quantity supplied respectively.

(i) Find the inverse demand and supply functions

Qd = 110-5P
5P = 110-Qd
P = 110-Qd/5

Qs = 6P
P = Qs/6

(ii) Find the equilibrium price and quantity

Solve simultaneously:
Qd = 110-5P
Qs = 6P

At equilibrium Qd = Qs

$110-5P = 6P$

Collect the terms
$-5P-6P = -110$
$11P = 110$
$P = 110/11$
$P = 10$

Solve for Q*
$Qd = Qs = 6P = 6(10) =$ **$60 = Q^*$**

3. **Demand and supply in a market are described by the equations**
$Qd = 120-8P$
$Qs = -6+4P$

a. Solve algebraically to find equilibrium P and Q

$Qd = Qs$
$120-8P =-6+4P$
$-8P-4P = -6-120$
$-12P = -126$
$12P = 126$
$P^* = 10.5$

$Qd = Qs = 120-8P = 120-8(10.5) = 120-84 =$ **$36 = Q^*$**

b. How would a per unit sales tax t affect this equilibrium and comment on how the tax is shared between producers and consumers

Supply price becomes P-t
Supply function becomes $Qs = -6+4(P-t)$
Solve for equilibrium
$Qd = Qs$
$120-8P = -6+4(P-t)$
$120-8P = -6+4P-4t$
$-8P-4P = -120-6-4t$
$-12P = -126-4t$
$12P = 126+4t$
$P = 10.5+4t/12$
$P = 10.5+t/3$

$Qd = Qs = 120-8(10.5+t/3) = Q^*$
$Q^* = 120-84-8t/3$
$Q^* = 36-8/3t$

The impact of the tax will therefore be to increase equilibrium price by 1/3 and reduce equilibrium quantity by 8/3. Since 1/3 of tax is passed on to the consumer the supplier pays 2/3 of the tax.

c. What is the equilibrium P and Q if the per unit tax is 4.5

P = 10.5+t/3
P = 10.5+4.5/3
P = 10.5+1.5
P = 12

Supplier gets 10.5-2/3t = 10.5-3 = 7.5

Q = 36-8/3t
Q = 36-8/3(4.5)
Q = 36-12
Q = 24

4. **At a price of €15, and an average income of €40, the demand for CDs was 36. When the price increased to €20, with income remaining unchanged at €40, the demand for CDs fell to 21. When income rose to €60, at the original price €15, demand rose to 40.**

i) Find the linear function which describes this demand behaviour

General Form: Qd = a+bP+cY

P = 15, Qd = 36, Y = 40
P = 15, Qd = 40, Y = 60
P = 20, Qd = 21, Y = 40

Eq1 36 = a+15b+40c
Eq2 40 = a+15b+60c
Eq3 21 = a+20b+40c
Solve Simultaneously

Eq1 36 = a+15b+40c
Eq2 40 = a+15b+60c
STEP 1
a = 36-15b-40c
a = 40-15b-60c
STEP 2
36-15b-40c = 40-15b-60c
STEP 3
-15b+15b-40c+60c = 40-36
20c = 4
c = 4/20 = 1/5
STEP 4
Eq1 36 = a+15b+40(1/5)
36 = a+15b+8
36-8 = a+15b
28 = a+15b

Eq3 21=a+20b+40(1/5)
21-8 = a+20b

13 = a+20b

STEP 1
Eq1' 28=a+15b
Eq2' 13=a+20b
a = 28-15b
a = 13-20b
STEP 2
28-15b = 13-20b
STEP 3
-15b+20b = 13-28
5b = -15
b = -3
STEP 4
a = 28-15b
a = 28-15(-3)
a = 28+45
a = 73

General Form
Qd = a+bP+cY
Qd = 73-3P+1/5Y

ii) **Given the supply function Qs = -7+2P find the equations which describe fully the comparative statics of the model.**
Qd = 73-3P+1/5Y
Qs = -7+2P
In equilibrium Qd = Qs
73-3P+1/5Y = -7+2P
-3P-2P = -7-73-1/5Y
5P = 80+1/5Y
P* = 16+1/25Y

Qd = Qs = -7+2P = -7+2(16+1/25Y) = -7+32+2/25Y = 25+2/25Y = Q*

iii) What would equilibrium price and quantity be if income was €50?

P* = 16+1/25Y = 16+1/25(50) = 16+2 = 18
Q* = 25+2/25Y = 25+2/25(50) = 25+4 = 29