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ECONOMETRICS AND TRUTH

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1. INTRODUCTION

> *"The only way to a position in which our science might give positive advice on a large scale to politicians and business men, leads through quantitative work. For as long as we are unable to put our arguments into figures, the voice of our science, although it may occasionally be able to dispel gross errors, will never be heard by practical men. They are, by instinct, econometricians all of them, in their distrust of anything not amenable to exact proof".*
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> From Joseph A Schumpeter: 'The Common Sense of Econometrics', *Econometrica*, 1 (1933), p. 12, as quoted in Hendry (1995).

Many people think that econometrics is not the most exciting of subjects. They are more interested in the results of an econometric analysis than in the means and ways of obtaining them Many people have a distrust of econometric analyses. I am not sure whether this has arisen from the failure of the extravagant expectations of the sixties or the seventies or from a basic misunderstanding of the probabilistic nature of the subject itself. Many will offer the excuse that the data are not good enough to support elaborate estimation techniques, complex models or rigorous techniques. The simple answer is that a solution to these problems demands the use of elaborate techniques. The availability of cheap powerful computer facilities has not only made this methodology available to a large number of users but has also acted as catalyst for its further development.

Perhaps econometricians, themselves, have failed to explain much of their recent work to those outside their speciality. Pagan and Wickens (1989) in a survey of econometric methods refer to the fragmentation of econometrics. As the subject grows in complexity it becomes more difficult even for a specialist econometrician to keep up with new developments. In recent years the programmes of the Econometric society have shown the diversity of the subject. On occasion there have been just short of twenty simultaneous sessions. Is it any wonder that econometricians specialising in one topic have problems communicating with others specialising in another let alone with non-econometricians?

In writing this paper I have tried to explain the problems that have led to this diversity, to outline the types of solutions being examined and to show their relevance to current quantitative economic analysis. I can not, in this paper, show you how to implement all the latest methods in your own work. My aim is to describe modern econometrics and in particular its application to macroeconomics. I will try to show you how quantitative work can benefit from the application of these techniques. Even if you have no interest in using the techniques I hope that this paper will explain terms such as "unit roots", "cointegration ", "chaos" etc. and increase your appreciation of much of modern applied econometric work

Econometrics is often defined as the application of mathematical, statistical and other quantitative methods to economics. The purpose of econometrics is to increase our understanding of quantitative relationships in economics and thus to improve policy analysis and advice. Quantification is an essential part of this work and must have a sound statistical base. Quantification based on invalid statistical advice is dangerous and takes away from rather than supports analysis based on good intuition and rhetoric. The main aim of this paper is a description of statistical and quantitative methods that have a practical applications in policy analysis.

When the idea of this paper was first suggested I was reading Richard Von Mises[1] book "*Probability, Statistics and Truth*" and I have taken my title from his. Both titles imply a contradiction between the ideas of econometrics or statistics and truth. We have all heard stock remarks such as: "There are three kinds of lies; lies dammed lies and statistics" or "Anything can be proved by figures" or even "All persons may contend their charming systems to defend". Such statements imply that conclusions drawn from statistical reasoning are at best uncertain and at worse misleading. Von Mises set out to show that despite the fact that *"a great deal of meaningless and unfounded talk is presented to the public in the name of statistics.... starting from statistical observations and applying to them a clear and precise concept of probability it is possible to arrive at conclusions which are just as reliable and 'truthful' and quite as practically useful as those obtained in any other exact science".*

It is almost seventy-five years since Von Mises wrote those words. In the field of economics we continue to hear the same disparaging remarks applied, in particular, to many applications of statistics to economics. We can all think of misuse of statistics, caused by misunderstanding or deliberate misrepresentation. It is often possible to subtly change the impact of data by presenting it in a particular kind of way or by presenting only part of the data. It is a great pity that many persons think that such conclusions come from what they think of as valid statistical analyses. We can often get well-meaning people quoting what they believe to be respectable techniques to support their personal beliefs. In extreme cases their method may be invalid, their analysis sensitive to the model and data used and their results reversible by minor changes in either. A proper use of the theories and methods outlined below may make econometrics more difficult to use and leave various questions unanswered but will definitely improve the quality and dependability of results.

From the early thirties to the late sixties or early seventies the main developments in econometrics were in the estimation of simultaneous equation models. In the mid-sixties this had developed to the stage that some people appeared to think that with some developments in computer power, further economic research and better data we might be able to allow a computer equipped with a suitable macro model to run a country or, at least, dictate economic policy. In the seventies it became apparent that these theories contained several large flaws. At the same time there were considerable developments in time series methods, led, no doubt, by the relative success of the Box-Jenkins methodology. In section 2, I give a brief selective account of these developments.

Most of the theory described in section 2 assumed that the data was stationary or could be made stationary by a suitable transformation. In particular there was no justification for including many types of non-stationary variables in OLS regressions. Section 3 sets out the history of the problem of spurious regressions and provides a description of what we now understand to be its cause and links the theories of spurious regressions and cointegration. The developments described in this section have had a profound impact on recent econometric methodology.

Recent developments in computer science have made vast amounts of computer power available to economists who now have the resources to go beyond the basic assumption of linearity inherent in many of their models. Section 4 examines some of the ways in which non-linearity has been introduced. At the end of this section there is a brief description of chaos theory and its implication for economics.

Sections 2, 3, and 4 assume that the relevant economic theory is known and that the data used measure the concepts they are supposed to measure. Section 5 describes how statistical theory can aid in the search for the "true" theory.

Section 6 contains a description of some recent developments in micro-computer software designed for econometric analysis.

Section 7 contains some concluding remarks and comparisons of the meaning of truth in economics and the natural sciences.

2. STRUCTURAL MODELS AND TIME SERIES ECONOMETRICS BEFORE COINTEGRATION

The principal difference between the application of statistics to economics and to other sciences arises from the non-experimental nature of economic data. The problem is well described in terms of the following parable - the source of which is unknown to me.

> *"An agricultural research professor wished to study the effects of a new fertiliser on the yields of a certain crop. His experimental design divided a field into plots and using a suitable randomisation process allocated seed and new fertiliser to one set of plots and seed and old fertiliser to the remainder. The crops grew and the new fertiliser yielded much more than the old. The results were seen as a great advance and various honours were heaped upon the researcher. Another professor decided to examine the results again. The field happened to contain a large number of trees and professor No. 2 found that all the good results had come from plots that were in the shade of the trees while the bad results were in the sunlight. He propounded an alternative theory which again was well received and various honours were heaped upon him. The world research body split into two. Those supporting professor No. 1 set up caricatures of the theories of professor 2 and proceeded to demolish these caricatures. The supporters of professor 2 behaved in a similar way towards the theories of professor 1. The parable goes on to relate how whole countries, sometimes at great cost to their citizens, adopted one of the theories. After a period, most people lost faith in the theories of both professors".*

In simple terms the design of the original experiment was bad. It appears that the new fertiliser was applied to the shaded areas. Thus, one cannot tell whether the improved returns are due to the new fertilisers or the shade. The experimenter should have randomised this effect and should be severely criticised for not doing so.

In economics we are often meet this kind of problem. By the very nature of our science our data is simply the history of the economy. As statisticians we have no say in the design[2] of the experiments that produce the data. In economics we are often confronted by two theories, both of which could have explained history but have different policy implications. This is the problem of observational equivalence. Statistical theory allows us to estimate a reduced form model that provides a value for each endogenous variable in terms of lagged endogenous variables (history) and current exogenous variables. However, This may not solve the problem. Various economic theories may lead to the same reduced form model.

A further non-statistical problem is that of identification. A simple example of this is the estimation of demand and supply functions. Suppose we have a dataset which contains the prices and quantities of a good such as corn traded in a market. We estimate the relationship between the quantity traded and its price. Does this relationship represent a Supply or a Demand curve? The answer is probably neither as the dataset is the intersections of a series of shifting Supply and Demand Schedules Now, regard the Demand curve as fixed and the supply curve as moving up and down due to fluctuations in some supply condition such as changing weather which does not have a significant effect on the demand for corn. The points of intersection in our dataset are all on the demand curve. The estimated relationship will be the Demand schedule. The effect of the weather on the Supply schedule is identifying the Demand schedule. This problem was tackled correctly by Tinbergen as early as 1930. He used what amounted to zero restrictions to identify models. Tinbergen's results and those of Frisch (1933b) did not attract a great deal of attention for nearly ten years. Their basic idea was that identification must be achieved by non-sample means (in the example above - the assumption that the weather did not effect the demand for corn). The mathematics are somewhat abstract and often the basic idea is lost in the formalism (rank and order conditions etc.). A model that is not identified is incomplete and does not make sense. It should not be used. I will not deal with identification at any length as the main results have been known for many years. Useful surveys are Fisher (1966) and more recently Hsiao (1983). Most intermediate to advanced econometric texts cover the topic adequately. In what follows I will assume that we have made sufficient use of non-sample information to ensure that the problems we are tackling are identified[3].

From the 40's to the late 60's/early 70's, the macro-econometric model was the focus of considerable research. During this period it appeared that the econometricians had solved all the identification and statistical estimation problems. With the advances in computer hardware and specialist econometric software one could estimate, solve and simulate large models. Some of these models involved hundreds and even thousands of equations.

Models were set up to provide forecasts of the economy at long horizons and to evaluate many kinds of policy scenarios. For a while great importance was attached to many of these models both by those who produced them and by those who used or indeed misused them. Many people thought that with some developments in computer power, further economic research, and better data we might be able to allow a computer equipped with a suitable econometric model to run a country or at least dictate economic policy.

From the early 70's things began to go wrong. Typically the problem was not that the models failed to predict the first oil crisis but that many of them failed to track events after the crisis. What then was wrong?

The models were built on the assumption that economic theory was known and was accurately represented by the equations of the model. The most that could be admitted was that the model was an approximation to the real world and not a very accurate representation at that. Each model must have involved choices between alternative theories such as Keynesian or Classical forms, market clearing or rationing, functional forms used in production functions, consumption functions, etc. The dynamic adjustment processes involved were not well understood and it is likely that such processes, if any, were over simplified. Much of the systems assumed that the economy was almost linear - a most heroic assumption. Many of the models took little or no account of the fact that the economy is continuously changing. A time trend often substituted for technical progress. In many cases the data used in the analysis did not correspond exactly with the economic variables modelled. In many cases the model did not use the best statistical practice. All macro-economic models in the early 70's were subject to some of these problems. Is it any wonder that the models did not account for the progress of the economy after the oil crisis.

To some extent these criticisms overstate the practical case against econometric modelling. With our improved knowledge of statistics many of the problems could be overcome. Our knowledge of economics is improving. We are no closer to a situation where a macro-econometric model will provide all answers. Lucas (1976) provided a serious theoretical criticism of policy evaluation using econometric models. He argued that the equations of an econometric model represented the optimising decision rules of economic agents who used the rational expectations hypothesis. These optimising decision rules were likely to change systematically when a change in policy impacted on the decisions of the agent. Thus, the coefficients in the usual consumption or production function may change as policy changes. To overcome this criticism we need to estimate the utility or production function or at least the parameters involved in these functions. These parameters are referred to as the deep structural parameters of the model.

To estimate the deep structural parameters we need to assume a form for, say, the utility function. The optimisation problem leads to a set of Euler equations that depend on deep structural parameters. These systems are generally non-linear. The appropriate estimation technique is Generalised Methods of Moments (GMM). Such systems have provided valuable insights into Monetary Theory, Consumption Theory and other areas of dynamic macroeconomic theory and Finance. See McCandless and Wallace (1991), Stokey and Lucas (1989), Sargent (1987), or Azariadis (1993) for details. There are considerable problems in using these models. In particular we do not know, and will never know, the exact form of the utility or production function that contains the deep parameters. The best we can do is to adopt some functional form that is mathematically tractable. Many of the models are representative agent models and may not aggregate to the level of the macroeconomy. Thus, the estimation of deep structural parameters may not lead to better econometric models.

This does not invalidate the Lucas critique which may be valid even in the absence of rational expectations. To evaluate policy we must have some idea of the effects of changes of policy on the parameters of our model. Consider, for example, a consumption function and imagine that there had been many changes of policy over the estimation period. If the parameters of the consumption function have been tested for stability and the tests had passed then we might reasonably assume that the Lucas critique was not important in this case. If there had been few changes in policy, we would not be able to make this conclusion and would need to be more cautious about our conclusions. A test of parameter stability is an indication of the relevance of the Lucas critique for the problems under examination.

Despite the many problems arising the rational expectations hypothesis has had a considerable impact on both economics and econometrics. Lucas and Sargent (1981) present most of the early applications. The survey material in Pesaran (1987b) covers many of the problems arising from the identification and estimation of rational expectations models.

In the late 60's/early 70's the "atheoretical" Box-Jenkins methodology posed another challenge to macro-econometrics. The earliest drafts of Box and Jenkins (1976) were produced in 1965 and published as technical reports of the Department of Statistics, University of Wisconsin and of the Department of Systems Engineering, University of Lancaster. The Box-Jenkins methodology is basically a univariate forecasting methodology in which a variable is forecast from its own history[4]

The methodology must have appeared somewhat unconventional to econometricians.

The aim of Box-Jenkins (1976) was

> *"..... to derive models possessing maximum simplicity and the minimum number of parameters consonant with representational adequacy.*
>
> *The obtaining of such models is important because:*
> *(1) They may tell us something about the nature of the system generating the time series;*
> *(2) They can be used for obtaining <u>optimal forecasts</u> of future values of the series;*
> *(3) When two or more related series are under study, the models can be extended to represent dynamic relationships between the series and hence to estimate transfer function models..."*

The analysis progressed in three steps:

1. Initial identification of model
2. Estimation of model
3. Evaluation of model

If at step 3 the model was inadequate its specification was amended and steps 2 and 3 repeated. This cycle of steps 2 and 3 were repeated until a satisfactory model was found.

In step 1, the initial action was to find a transformation (logs, square roots first differences and/or seasonal differences) that would make the series stationary. (We will be saying a lot about stationarity later.) Graphical methods were an important part of the process. It was often possible to use other information to improve forecasts. At that time the estimation of these systems required significant computer resources. To-day most econometric packages perform a full maximum likelihood estimation with a single simple instruction. On the third stage statistical tests were important but did not always have the last word. Often if the sample was small a more parsimonious system might be chosen even though the statistics for a larger model were superior.

To many econometricians it was a most unpleasant surprise that these naive forecasting methods produced better forecasts in many cases than the most elaborate macroeonometric models. The literature on this topic is so large that I will make no attempt to survey it at length. Fildes (1985) looks at sixty studies of comparative accuracy of econometric (causal) and extrapolative models. According to his summary 60% of the almost 400 series, considered in the 60 studies, were forecast better by econometric (causal) methods. Likewise the extrapolative methods forecast better in almost 40% of cases. The result is at best inconclusive.

Granger and Newbold (1986) addressed the same problem. They report on a study by Cooper (1972) who examined seven previously specified models of the U.S. economy, designated Fried-Taubman, Fromm, Klein, Liu, OBE, Wharton-EFU and Goldfield. They fitted each model with a sample of 48 quarterly observations. They also fitted a naive univariate auto-regressive model to 33 exogenous variables over the same sample period. The orders of the autoregressions were determined by an automatic optimising procedure up to a maximum of eight lags. The econometric forecasts were obtained by solving the models with actual future levels of the exogenous variables. The number of times each method performed best was:

Fromm	7
Liu	1
Klein	1
OBE	2
Whatron-EFU	4
Friend-Taubman	0
Goldfield	0
Naive	18
	33

Granger and Newbold (1986) point out that a detailed examination of Cooper's findings reveals, a position even less complimentary to the econometric models. Every model examined was outperformed, under the mean-squared-error criterion on a substantial majority of occasions by the autoregressive predictor. Moreover, very frequently the naive forecasts were better than the model forecasts by very substantial margins indeed. While these results do not augur well for macro-econometric forecasts other research in the early 70's showed that a combined time series and model forecast could improve on the time series forecast alone, indicating that the model forecasts were processing some information better than the time series approach.

Two points must be added to the above analysis. No attempts were made to refine the results of the econometric models. When macro-econometric models are "adjusted" by the user, to include his own knowledge of the economy their performance is greatly improved. The relative performance of the models is also better at longer horizons. Recent analyses tend to imply that the performance of models has improved in recent years. Fagan and Fell (1991) provide a good survey in a paper read to this society in February 1991. Honohan (1991) and Fitzgerald (1991) in comments on this paper provide a stout defence of the Hermes model.

The question that should be asked is not whether naive models or full structural models are better but rather which should be used for the problem in hand. Zellner and Palm (1974) show that there is a certain correspondence between the naive univariate models and full structural models.

The choice of model depends on the possible use of the model, the data available and the resources available to complete the analysis. Forecasting is perfectly feasible when the form of ARMA process is known and the parameters estimated. The richer dynamics included in the ARMA process may lead to better forecasts. An ARMA model can not be used for control or structural analysis. Control requires, at least, a classification of variables as exogenous or endogenous. Structural analysis also requires identifying restrictions. ARMA methods can play a significant part in the testing of a structural model. When the forecasts are considerably better in one than the other there are indications that the other may be misspecified and may be improved.

3. SPURIOUS[5] REGRESSIONS AND COINTEGRATION

The idea of a spurious regression in econometrics is not new. Pesaran (1987a) quotes from Hooker (1901) who was clearly aware of the problem

> *"The application of the theory of correlation to economic phenomena frequently presents many difficulties, more especially where the element of time is involved; and it by no means follows as a matter of course that a high correlation coefficient is a proof of causal connection between any two variables, or that a low coefficient is to be interpreted as demonstrating the absence of such connection"*

Hooker in this and other analyses experimented with time-series data to try to separate long-term secular movements, periodic movements and short-term oscillations. He also advocated removing the effects of periodic (cyclical or seasonal) by measuring deviations from a moving average. At the time, opinion was that the high correlation was due to the influence of the 'time factor' taken as a proxy for some other variable or variables which directly or indirectly caused the two variables to move together.

The title of Yule's 1925 Presidential address to the Royal Statistical Society (Yule(1926)) is worth repeating

> *"Why do we sometimes get nonsense correlations between time series? - a study in sampling and the nature of time series"*

Yule defined the problem as follows.

> *"It is fairly familiar knowledge that we sometimes obtain between quantities varying with the time (time-variables) quite high correlations to which we can not attach any physical significance whatever, although under the ordinary test correlation would be held to be certainly 'significant'. As the occurrence of such 'nonsense correlations' makes one mistrust the serious arguments that are sometimes put forward on the basis of correlations between time series ..... it is important to clear up the problem how they arise and in what special cases"*

Yule rejected the simple "time variable" effect as the cause of spurious regressions. He pointed to two problems. Firstly sampling theory assumed that samples were drawn at random from a population. Time series were consecutive and not random.

Secondly economic data are serially correlated and thus not independent. Thus, because of these departures from the assumptions required for a random sample, he argued that

> *".. the usual conceptions to which we are accustomed fail totally and entirely to apply".*

Strong criticism, indeed, but these problems were, in many cases, either assumed away or ignored over the next forty or so years. Granger and Newbold (1974), in an influential paper, extended our understanding of spurious regressions. They gave a simple theoretical analysis that showed how the regression of one random walk or another would give rise to spurious results. They also produced Monte Carlo studies to show the magnitude of the problem. We shall refer again to this work but first a practical example, using Irish data, and taken from Frain (1993). The variables involved were

IR3MTH	Irish 3 month interest rate
GER3MTH	German 3 month interest rate
IRSTG	Irish pound Sterling Exchange Rate
DMIR	DM Irish pound Exchange Rate
IRSPREAD	Spread between the long gilt yield and IR3MTH
CONST	Constant Term
TREND	Trend.

The data were end week and cover the period 8 May 1987 to 1 January 1993 and were taken from internal records in the Central Bank.

Table 1 gives the results of the regression. The t-statistics are very large and significant if judged by 'normal' standards. The value of R^2 is .96328 and is very close to 1. There are three serious problems in this analysis. First possible serial correlation in the residuals will, for standard text book reasons (Johnson (1984)) (Section 8.2), give rise to inconsistent estimates of the standard errors of the coefficients. In the table I have omitted any reference to a Durbin-Watson statistic of 0.46875 that indicates that we should, at least, proceed carefully.

Table 1 Estimate of Interest Rate Model			
Dependent variable is IR3MTH			
296 observations used for estimation from 8/5/87 to 1/1/93			
Regressor	**Coefficient**	**Standard Error**	**T-Ratio [Prob]**
GER3MTH	-47.3897	6.3759	-12.8449 [.000]
UK3MTH	-0.1223	0.0181	-6.7394 [.000]
IRSTG	18.3961	0.8112	22.6786 [.000]
DMIR	15.4165	2.3803	6.4767 [.000]
IRSPREAD	0.7881	0.0282	27.9228 [.000]
CONST	-47.3897	6.3759	-7.4326 [.000]
TREND	-0.0171	0.0013	-12.8449 [.000]
R-Squared	0.96	F-statistic F (6,289)	1263.7 [.000]
R-Bar-Squared	0.96	S.E. of Regression	.37353
RSS	40.32	Mean Dept. Var.	10.30
St. Dev Dept Var	1.93	Max Log-lik	-129.98

Secondly we can not assume that all the right-hand side variables are exogenous. If they are not strictly exogenous the correlation between the error term and any endogenous variables on the right hand side of the equation will lead to bias in the estimates of the coefficients. If they are not weakly exogenous in the sense of Engle Hendry and Richard (1983) inference on relevant parameters is not valid. Last, but probably most important, non-stationarity in the data implies that the regression coefficients may be spurious, in the sense of Granger and Newbold (1974) or Philips (1986).

Further analyses reveal that the regression, despite the attractive t-statistics and R-squared, fails to pass many misspecification tests and would in effect be rejected on these grounds. To understand the problem better we first look at some basic definitions in the statistical theory of time series. The elements of a time series may be denoted by:

$$X_1, X_2, ...X_t, ...$$

The mean and variance of the time series at time t are given by

$$\mu_t = E(X_t) \quad : \qquad \sigma^2_t = E[(X_t - \mu_t)^2]$$

respectively and the covariance of X_t X_s by

$$Cov(X_t X_s) = E[(X_t - \mu_t)(X_s - \mu_s)] = \lambda_{ts}$$

As we have only one observation at each period in time we can not estimate μ_t σ^2_t and λ_{ts} for all t and s. To proceed we need some additional simplifying assumption such as stationarity. A series is second-order stationary if

$$\mu_t = \mu \qquad t = 1, 2, ...$$

$$\sigma^2_t = \sigma^2 \qquad t = 1, 2, ...$$

$$\lambda_{ts} = \lambda_{t-s} \qquad t, s = 1, 2, ...$$

i.e. the mean, variances and covariances of the series are independent of time.

A series is strictly stationary if the joint distribution of $[X_1, X_2 .....X_m]$ is the same as that of $[X_{1+t}, X_{2+t}, ...X_{m+t}]$ for all m and t. If $[X_1, X_2.....X_m]$ is multivariate normal for all n, then second order stationarity implies strict stationarity. Many series may be stationary after a deterministic trend (or other deterministic component) has been removed. Such series are said to be trend reverting.

In elementary regression theory the explanatory variables are taken as given or fixed. The theory may be extended to include stochastic variables if they are well behaved. Series that are stationary or trend-reverting may be included, subject to certain regularity conditions. Judge et al. (1985), Chapter 5, deal with the inclusion of trend reverting and stochastic variables in regressions.

The theory of non-stationary processes is important in economics as it is an implication of models of optimising rational agents. Typical examples include financial market variables such as futures contracts (Samuelson (1965)), stock prices

(Samuelson (1973)), dividends (Kleidon (1986)), spot and forward exchange rates (Meese and Singleton (1983)) and aggregate consumption (Hall (1978)).

Nelson and Plosser (1982) applied statistical tests to 14 annual US time series and were able to reject stationarity in all but one. This analysis has been refined in various studies since then and various items have been added or deleted from the list of non-stationary series from time to time. Kwiatowski et al (1992) contains a recent analysis of these data. Thus, the conclusion drawn from both theory and statistical testing is that many processes are non-stationary and have a random walk component. Such a conclusion has important consequences for business cycle theories that argue that business cycles are transitory deviations from a stable path.

X_t is a random walk if

$$X_t = X_{t-1} + \varepsilon_t$$

where ε_t are uncorrelated variables with zero mean and constant variance. Errors or shocks in a random walk are persistent - once disturbed it shows no tendency to return to its original value. A random walk X_t is not stationary as its variance increases with time. However it's first difference

$$\Delta X_t = X_t - X_{t-1} = \varepsilon_t$$

is stationary. Variables that are not stationary in levels but are stationary in first differences are said to be integrated of order 1 and denoted I(1). The effect of including such variables in regressions has been the subject of considerable research in recent years[6]. Granger and Newbold (1974, 1986) show by means of simulation tests that the usual significance tests are invalid. Philips (1986) derives an asymptotic theory for regressions involving general integrated processes. He shows that, in certain circumstances, the t-statistics do not have a limiting distribution but diverge as sample size $T \rightarrow \infty$. He also shows that the Durbin Watson statistic converges to zero while the regression R^2 has a non-degenerate limiting distribution as $T \rightarrow \infty$. This is a fair description of our sample regression above. We can illustrate the significance of these results by means of the following simulation exercises that mimic this regression.

In the initial simulation 6 time series Y, X1, X2, X3, X4, X5 of independent normal random variables with zero mean and unit variance were drawn. The length of each time series was 296 observations to match the analysis in Table 1[7]. To simulate the type of result derived in Table 1 Y was then regressed on X1, X2, X3, X4, X5, a constant and a linear trend. An F test of the joint significance of the coefficients in the regression was carried out. Given the way the sample was generated the hypothesis of zero coefficients should have been accepted. The experiment was repeated 1000 times. At the 5% and 1% levels the hypothesis was rejected 50 and

12 times respectively. These results are completely in line with theory. This result is fully in accord with statistical theory.

The second experiment involved the same data sets but they were integrated to random walks before the regressions were carried out i.e. new (integrated) variables YI, X1I, X2I, X3I, X4I and X5I where

$$YI_t = YI_{t-1} + Y_t \qquad t = 2,3.....$$

$$YI_1 = Y_1$$

$$X1I_t = X1I_{t-1} + X1_t \qquad t = 2,3.....$$

$$X1I_1 = X1_1$$

and similarly for the other 4 integrated variables. These new variables are random walks and are therefore I(1). We now regress YI on X1I, X2I, X3I, X4I, X5I, a constant and a linear trend and carry out the same F test as before. When the experiment was repeated 1000 times all regressions contain significant coefficients at both the 5% and 1% level despite the fact that all the variables are independent. Thus, the F-tests and t-statistics lead to spurious results in all 1000 replications. It is interesting to look at the distribution of the 70 t-statistics in the first 10 replications.

T-statistics	Frequency
> 20	2
10 to 20	8
2 to 10	40
< 2	20

Thus, 50 out of a possible 70 coefficients have significant t-statistics. This clearly shows the spurious nature of the t-statistics in these cases.

The sample size in these simulations is larger than one would expect in an econometric analysis. The exercise was repeated for a reduced sample size of 50. For the stationary random variables 45 and 14 replications from 1000 showed significant coefficients at the 5% and 1% levels respectively. Again this is a reasonable result and verifies the validity of OLS methodology in the presence of stationary stochastic variables. However, when the variables are transformed to random walks and the analysis repeated as before 999 and 997 of the replications show significant coefficients at the 5% and 1% levels respectively. Note that the spurious regression problem is slightly worse in the larger sample - a feature that is obvious when you think about it.

Philips (1986) explained that the regression

$$Y_t = \beta_o + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{3t} + \mu_t$$

is spurious when Y_t X_{1t} X_{2t} X_{3t} are I(1) and we can not find β_0, β_1, β_2 and β_3 that will make

$$\mu_t = Y_t - \beta_0 - \beta_1 X_{1t} - \beta_2 X_{2t} - \beta_3 X_{3t}$$

stationary[8]. Hamilton (1994), Chapter 18 lists three possible approaches to curing the problem of spurious regressions.

The first is to add lags of both dependent and independent variables to the regression.

$$Y_t = \beta_0 + \varphi Y_{t-1} + \beta_1 X_{1t} + \gamma_1 X_{1t-1} + \beta_2 X_{2t} + \gamma_2 X_{2t-1} + \beta_3 X_{3t} + \gamma_3 X_{3t-1} + \mu_t$$

There exist values of the coefficients namely $\varphi = 1$ and $\beta_1 = \beta_2 = \beta_3 = \gamma_1 = \gamma_2 = \gamma_3 = 0$ such that μ_t is stationary. OLS estimates of this equation yield consistent estimates of all the parameters. The t-statistics for the βs and γs all have asymptotic N(0, 1) distributions. The F test for the joint null hypothesis that β and γ are zero has a non standard limiting distribution. Thus, including lags of all variables solves many of the problems associated with spurious regressions but tests of some hypothesis will have non standard distributions, even asymptotically. We must take care in interpreting the coefficients in such a regression.

The second approach is to difference the data when all variables become I(0). In this case inference is valid but long term information, if any, is lost. If the regression is spurious there is probably no long run information.

The third approach is to estimate the basic equation using a Cochrane-Orcutt approach. Blough (1992) has shown that this is asymptotically equivalent to the regression in first differences. Otherwise this approach has no advantages and may in certain circumstances cause problems.

We are left with the alternative where we can find β_1 β_2 β_3 such that μ_t is stationary. In this case each of the variables Y_t , X_{1t} , X_{3t} , and X_{3t} are I(1). In general any linear combination of I(1) variables is also I(1) but in this particular case the linear combination

$$Y - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_3$$

is stationary. In this case the variables are said to be cointegrated. As an example, consider the following. If appropriate definitions of Income (Y_t) and Consumption

(C_t) are used various theories predict that the ratio C_t/Y_t will tend to a constant. Using lower case letters to denote natural logarithms we may write this relationship as

$$c_t - y_t = k + \varepsilon_t$$

where ε_t is the effect of a shock to the system. If the system tends to revert to the constant k then ε_t will be a stationary random variable. At the same time ε_t and Y_t may be non stationary. Reasoning such as this is the basis for the error correction mechanisms such as those used by Sargan (1964), Davidson et al. (1978) and in many recent papers. A simple example of an equation involving an error correction mechanism is

$$\Delta c_t = \alpha\Delta c_{t-1} + \beta\Delta y_t + \gamma\Delta y_{t-1} + \delta(y_{t-1} - c_{t-1})$$

If c_t , y_t are I(1) and $y_t - c_t$ is I(0) then all the variables in this expression are I(0) and there is no need to worry about spurious regressions[9]. It is clear that the long-run equilibrium solution of this equation is

$$y_t = c_t$$

(Put all first differences equal to zero.) Thus, unlike our equation in first differences that contains no long run information the expression involving an error correction mechanism may contain long run information. If there are more than two variables in the analysis, say p, there may be n (<p) cointegrating relationships (linearly independent stationary linear combinations). Any number of these cointegrating relationships may appear in any of the equations in the system.

Testing for stationarity/non-stationarity and cointegration plays an important part in the analysis of any dataset and in particular in one where economic theory indicates that a time series is or may be non-stationary. Here, however, a serious problem arises.

If we have a particular finite set of observations from a non-stationary time series we can approximate it as closely as we like by a stationary time series. Similarly a particular finite observation from a stationary time series may be approximated as close as we like by a non-stationary time series. Thus, there is a certain observational equivalence between non-stationary series and stationary series. Essentially it is impossible to distinguish between non-stationary and stationary series with a finite sample[10]. If we have a sample of size T the information in the sample about effects over a period of length of the same magnitude as or longer than T must be very weak. However, we can inquire about the persistence over shorter periods and if over such shorter periods the process is very close to being non-stationary.

We can also arrive at testable hypotheses if we restrict our processes to the family of AR(p), MA(q) or ARMA(p, q) or some other parsimonious processes.

In these cases we still have a local problem such as distinguishing between local alternatives such as $\varphi = 1$ and $\varphi = .99$ in the AR(1) process

$$x_t = \varphi x_{t-1} + \varepsilon_t$$

This local problem could, in theory, be solved by, increasing the sample size and is no different from the usual problem that arises in a test of any hypothesis. The observational equivalence problem can not be overcome. The classes of stationary and non-stationary processes are to large. The problem is analogous to estimating seasonal patterns with too few seasonal cycles. Tests of unit roots must be viewed in this light.

A further problem in unit root tests arises from structural changes and other changes in regime. Perron (1989, 1994) shows that the existence of structural breaks biases the conclusions of unit root tests towards the acceptance of non-stationarity. He also provides tests for structural breaks in the presence of unit roots. Perron's conclusions will no doubt be examined at greater length in the future.

Ghysels and Perron (1992) examine the effect of seasonal adjustment filters such as X11 on unit root tests. They find that such procedures reduce the power of the tests considerably. Ericsson et al.(1994) find that inference with seasonally adjusted data may be problematic as the adjustment process may alter the dynamic process and exogeneity status of the data. It is recommended that one should use unadjusted series and use the methods described in Hylleberg et al (1990) and Osborn et al. (1988) to deal with problems of seasonality and unit roots.

Despite the difficulties involved, it can be important to test variables or combinations of variables for unit roots. We have already mentioned some of the consequences of efficient markets and persistence in economics. Processes that have a root which is close to 1 are often known as near integrated processes. Banerjee et al. (1993) (Section 3.6) give an account of Phillip's (1987) account of the asymptotic distributions of near-integrated processes. Their results suggest that when these processes are modelled as unit root processes the small sample behaviour of the resulting estimates is better than that of the standard stationary analysis. Similarly Evans and Savin (1981) examine the limiting distribution of an AR(1) process. For $\rho<1$ the OLS estimate of ρ has a standard asymptotic normal or t-distribution. However, ρ near to 1 the "unit root" distributions may be a better finite sample distribution. The initial tests for unit roots were variants of the basic Dickey Fuller test. This test is based on the regression

$$x_t = \rho x_{t-1} + \varepsilon_t$$

If x_t is I(1), the null hypothesis, the true value of ρ is 1 and we expect an estimate of ρ close to 1. Alternatively if we run the regression

$$\Delta x_t = \lambda x_{t-1} + \varepsilon_t$$

we would expect a value of λ close to 0. If we calculate the t-statistics for zero λ we should be able to base a test of $\lambda = 0$ on this statistic. However, the distribution of this statistic does not have the usual t-distribution but follows a distribution originally tabulated by Fuller (1976).

To summarise we let our maintained hypothesis be

H_0 $\lambda = 0$ (unit root)

against an alternative

H_1 $\lambda < 0$ (stationarity)

and reject the unit root for sufficiently small values of the t-statistic. Note that the alternative is an AR(1) process with parameter less than one (i.e. a stationary AR(1) process).

In effect there are four such tests (ε_t is a white noise).

	Test Regression	H_0 (True Model)
1.	$\Delta X_t = \lambda X_{t-1} + \varepsilon_t$	$\Delta X_t = \varepsilon_t$
2.	$\Delta X_t = \alpha_1 + \lambda X_{t-1} + \varepsilon_t$	$\Delta X_t = \varepsilon_t$
3.	$\Delta X_t = \alpha_1 + \lambda X_{t-1} + \varepsilon_t$	$\Delta X_t = \alpha_1 + \varepsilon_t$
4.	$\Delta X_t = \alpha_0 t + \alpha_1 + \lambda X_{t-1} + \varepsilon_t$	$\Delta X_t = \alpha_1 + \varepsilon_t$

The t-statistics for $\lambda = 0$ yield in 1, 2 and 4 the test statistics that Fuller calls τ τ_μ and τ_τ respectively. The t-statistic for $\lambda = 0$ in 3 has an asymptotic standard normal distribution. This latter process is, in my opinion, not that important in economics. Moreover, Banerjee et al (1993 p. 105) suggest that, in finite samples, the Dickey-Fuller distributions may be a better approximation than the normal distribution. In 1, 2 and 4 the joint distributions of α_0 α_1 and λ have non-standard distribution. It is possible to formulate joint hypotheses about α_0, α_1 and λ. Critical values are given in Dickey and Fuller (1981) and these have been reproduced in several books.

Critical values for these statistics are not affected by heteroskedasticity in the error term. The regressions must be amended to allow for serial correlation. The presence of serial correlation may be thought of as implying that we are using the 'wrong' null and alternative hypotheses. Suppose that we assume that the first difference follows an AR(p) process. Augmented Dickey-Fuller (ADF) tests are then appropriate. In an ADF test the regressions above are augmented by lags of ΔX_t.

	Test Regression	H_0 (True Model)
5.	$\Delta X_t = \lambda X_{t-1} + \sum_{j=1}^{p} \varphi_j \Delta X_{t-j} + \varepsilon_t$	same with $\lambda = 0$
6.	$\Delta X_t = \alpha_1 + \lambda X_{t-1} + \sum_{j=1}^{p} \varphi_j \Delta X_{t-j} + \varepsilon_t$	same with $\alpha_1 = \lambda = 0$
7.	$\Delta X_t = \alpha_1 + \lambda X_{t-1} + \sum_{j=1}^{p} \varphi_j \Delta X_{t-j} + \varepsilon_t$	same with $\lambda = 0$
8.	$\Delta X_t = \alpha_0 t + \alpha_1 + \lambda X_{t-1} + \sum_{j=1}^{p} \varphi_j \Delta X_{t-j} + \varepsilon_j$	same with $\lambda = \alpha_0 = 0$

In 5, 6 and 8 the t-statistics for $\lambda = 0$ have the same τ, τ_μ and τ_τ distributions as those of the unaugmented regressions. The t-statistics for $\varphi_j = 0$ have standard distributions in all the regressions. Note that the joint distributions of α_0 α_1 and λ may have non-standard distributions as in the unaugmented case.

The augmented Dickey-Fuller test assumes knowledge of p, the order of the autoregressive system. In general, this is not known and must be estimated. There are various alternative ways of doing this. One procedure estimates p as the minimum number of lags that must be added to the basic system to ensure that a Q test for no serial correlation is passed. A second method starts with a maximum number of lags (say n) and reduces this one by one lag at each stepof the procedure. At the k^{th} step we have p-k lags. The order chosen for the model is the minimum value of k for which the reductions from p to n-k, n-1 to n-k, ... , n-k+1 to n-k are not significant. Alternatively the required lag can be determined using either Akaike (1969) AIC criterion or Schwarz (1978) BIC criterion or similar. These procedures are likely to give a range of values for p and one should look at the sensitivity of the ADF statistic to the range of values.

Philips (1987) and Philips and Perron (1988) proposed an alternative way of dealing with autocorrelated variables. Their method is more general and may be considered as an extension to testing within an ARMA class of functions. They calculate the same regressions as in the Dickey Fuller case but adjust the test statistics using non-parametric methods to take account of more general autocorrelation and heteroscedasticity. Said-Dickey (1985) Augmented Dickey Fuller tests also provide a valid test procedure for general ARIMA (p, i, q) processes.

Various other tests for unit roots have been proposed and are surveyed in Stock (1993).

Alternatively Kwiatkowski et al (1992) and Park, Ouliaris and Choi (1988) have proposed tests for which the null is I(0) and the alternative I(1). These tests are a useful supplement to the standard tests particularly as the power of the original tests may is low.

It should be clear by now that there is no easy answer to the question as to which unit root test is best. It entirely depends on the purpose of the analysis, the likely sensitivity of the results to the outcome of the test and other available economic information. If intercepts and trend terms are included in (Augmented) Dickey Fuller tests when they are not required efficiency is reduced. Excluding such terms when they are required reduces the asymptotic power of the tests to zero. Holden and Perman (1994) and Enders (1995) propose elaborate schemes of sequential tests to overcome these problems. Unless economic theory indicated otherwise I would tend to use an (Augmented) Dickey Fuller test with trend and constant included and the corresponding Philips-Perron test. I would also use the Kwiatkowski test to confirm these results. An example of this simplified procedure is Frain (1993).

In general if X_1 and X_2 are I(1) any linear combination of X_1 and X_2 is I(1) and any regression of X_1 on X_2 is spurious. If we can find a β such that $X_1 + \beta X_2$ is I(0) then X_1 and X_2 are said to be co-integrated and the regression of X_1 on X_2 is not spurious. The concept of cointegration is thus of great importance as it indicates when a regression involving I(1) variables is not spurious.

I have concentrated on showing the effects of unit roots on econometric practice. Their presence requires different inference procedures. We should not forget their economic implications. The absence of unit roots (i.e. stationarity) implies that the economic variable will, in the long run, revert to the original equilibrium path after a shock has hit the system. The existence of a unit root implies that the variable will not return to the original equilibrium path but will move to a new path. If you model a variable as stationary you are imposing a return to the original equilibrium path. In the circumstances, inference which depends on this return to equilibrium is problematic.

The concept of integration and cointegration can be generalised to more than two levels and to higher levels of integration as follows. X is integrated of order d denoted I(d) if it is stationary when differenced d times and non-stationary when differenced d-1 times. If X and Y are I(d) and we can find a β such that $X-\beta Y$ is I(d-p) then X and Y are cointegrated of order p.

We will continue for now with I(1) variables. Tests for cointegration are easily seen to be unit root tests on the residuals of certain regressions. To test if the I(1) variables $Y_1\ X_1 \ldots X_p$ are cointegrated we complete one of the following regressions

$$Y_t = \alpha_1 + \lambda_1 X_{12} + \lambda_2 X_{2t} + \ldots + \lambda_p X_{pt}, + \delta_{\mu t}$$

$$Y_t = \alpha_0 t + \alpha_1 + \lambda_1 X_{12} + \lambda_2 X_{2t} + \ldots + \lambda_p X_{pt} + \delta_{\pi}$$

The residuals

$$\delta_{\mu t} = Y_t - \hat{\alpha}_1 - \hat{\gamma}_1 X_{1t} - \hat{\gamma}_2 X_{2t} - \cdots\cdots - \hat{\gamma}_p X_{pt}$$
$$\delta_{\pi} = Y_t - \hat{\alpha}_1 t - \hat{\alpha}_1 - \hat{\gamma}_1 X_{1t} - \hat{\gamma}_2 X_{2t} - \cdots\cdots - \hat{\gamma}_p X_{pt}$$

are calculated and the Dickey Fuller regressions

$$\Delta\delta_t = \lambda\delta_{t-1} + \varepsilon_t$$

possibly augmented by lags of $\Delta\delta_t$ to give an (A)DF statistic for $\lambda = 0$. This statistic follows neither the t-statistic nor the Dickey Fuller Distribution used in testing for unit roots above. Its distribution also depends on whether δ_μ or δ_τ is used to derive the test statistic and on the number of variables in the original regression.

Philips and Ouliaris (1990) show that a version of the Philips Perron test, described above, performs well as a residual test for cointegration.

Leybourne and McCabe (1993) give a test for cointegration in which the null is cointegration.

Suppose we have time series X_t and Y_t and we wish to regress Y_t on X_t. Then:

1. if X_t and Y_t are both stationary the regression is valid subject to the usual conditions.

2. if X_t and Y_t are both I(1) and the residual in the regression is not stationary then the regression is spurious.

3. if X_t and Y_t are both I(1) and the residual in the regression is stationary (i.e. X_t and Y_t are cointegrated and the regression is valid)

4. if X_t and Y_t are integrated of different order then any regression is completely meaningless.

If there are three variables in the regression, versions of 1, 2 and 3 still apply. A variant of 4 applies but we must be cautious. Suppose X_t and Y_t were I(2) and we could find a β such that $X_t + \beta Y_t$ was I(1). Suppose that Z_t was I(1) and we could find γ such that $X_t + \beta Y_t + \gamma Z_t$ was I(0) then a regression of X_t on Y_t and Z_t would make sense. In such a case there must be a good economic reason for such a regression.

When more than two variables are involved we are faced with an additional problem. There may be more than one cointegrating relationship - i.e. if $X_1 \dots X_k$ are k time series, all of which are I(1) we can find

$\beta_1 \dots \beta_k$ and

$\gamma_1 \dots \gamma_k$

where the β's are not a constant multiple of the γ's

such that $\Sigma\beta_i X_i$ and $\Sigma\gamma_i X_i$ are both stationary. In such cases there could be as many as k-1 such stationary combinations (different cointegrating relationships).

When there is only one cointegrating relationship estimation and testing is often completed using the method recommended in Engle and Granger (1987). To illustrate this methodology we will assume that we have a simple bivariate system involving wages w and prices p, where both variables are expressed in logs and are I(1). We are interested in the VAR(2) system[11].

$$w_t = \alpha_{11} w_{t-1} + \alpha_{12} w_{t-2} + \alpha_{21} p_{t-1} + \alpha_{22} p_{t-2} + \varepsilon_t$$

$$p_t = \beta_{11} w_{t-1} + \beta_{12} w_{t-1} + \beta_{21} p_{t-1} + \beta_{22} p_{t-2} + \varepsilon_t$$

If w_t and p_t are cointegrated (i.e. $w_t - \beta p_t$ is stationary) the Granger representation theorem shows that the equations can be put in the form

$$\Delta w_t = \gamma_1 (w - \beta p)_{t-1} + \delta_{11} \Delta w_{t-1} + \delta_{12} \Delta p_{t-1} + \varepsilon_1$$

$$\Delta p_t = \gamma_2 (w - \beta p)_{t-1} + \delta_{21} \Delta w_{t-1} + \delta_{22} \Delta p_{t-1} + \varepsilon_2$$

If β is known these equations can be estimated by OLS. If β is not known Engle and Granger propose a two step method.

1. Estimate the regression of w on p (using OLS) to obtain an estimate of $\hat{\beta}$ of β. Calculate the residuals of this regression say η_t

2. γ_1 γ_2 δ_{11} ... δ_{22} are estimated by replacing the term $(w - \beta p)_{t-1}$ by η_{t-1} and estimate the regression using OLS.

The properties of this estimator are given in Theorem 2 of Engle and Granger

> "Theorem 2: The two step estimator of a single equation of an error correction system obtained by taking $\hat{\alpha}$ from (4.5)[12] as the true value, will have the same limiting distribution as the maximum likelihood estimator using the true value of α. Least squares estimators will be consistent estimators of the true standard errors".

It should be noted that $\hat{\beta}$ in this equation does not follow a standard distribution and its significance can not be tested using the usual t-statistics. A summary of extensions to these results is in Hamilton (1994 section 19.3).

This estimate of $\hat{\beta}$ may be subject to some small sample bias and Banerjee et al. (1986) use a Monte Carlo type analysis to confirm this. This problem is data dependent. It is more serious for large high frequency daily samples than for smaller series of annual data covering a longer time period. In bivariate regressions a high R^2 indicates low bias.

The economic meaning of the wage/price system can be seen from a consideration of the equations in differences. The long-run solution is found by equating all differences to zero and looking at the resulting equation

$$w = \beta p$$

which represents the long-run equilibrium of the system. The dynamics of the system are such that a part of the disequilibrium in one period is corrected in the next period by an interaction with both w and p. The remaining short run adjustments are through the lags of the difference terms. Thus, the system allows us to model

non-stationary variables

Long run equilibrium between non-stationary variables

short term dynamics.

Such a system will not in general be in equilibrium and temporary shocks may (or may not) have permanent effects.

When there are more than two time series and possibly more than one cointegrating vector the above methods do not always hold. The principal method for these cases is that proposed and worked in a series of papers by Johansen (1988, 1991a, 1991b, 1992a, 1992b, 1992c, 1994a, 1994b,) Johansen and Juselius (1990,1992, 1994) and Juselius (1992, 1994a, 1994b).

To illustrate these procedures suppose we have a four variable model involving output y, real money m, consumption c and prices p, all variables being measured in logs. As before assume that the system is a VAR (2) system. A typical equation in the VAR system being

$$y_t = \gamma_{11} y_{t-1} + \gamma_{12} y_{t-1} + \gamma_{21} m_{t-1} + \gamma_{22} m_{t-2} + \gamma_{31} c_{t-1} + \gamma_{32} c_{t-2} + \gamma_{41} p_{t-1} + \gamma_{42} p_{t-2} + \varepsilon_t$$

and three other simultaneous equations for m_t c_t and p_t. If y_t m_t c_t and p_t are I(1) the regression may be spurious. The Johansen procedure

1. Estimates the number of cointegrating relationships

2. Estimates the parameters of the cointegrating systems.

Suppose we find two cointegrating relationships say

$$m_t - p_t - 0.99y_t$$

$$c_t - 0.75\, y_t$$

we can rewrite our equations in the form

$$\Delta y_t = \alpha_1 (m_t - p_t - 0.99\, y_t) + \alpha_2 (c_t - 0.75\, y_t) + \mu_{11} \Delta y_{t-1} + \mu_{12} \Delta m_{t-1} + \mu_{13} \Delta c_{t-1} + \mu_{14} \Delta p_{t-1}$$

and similar expressions for Δm_t Δc_t and Δp_t.

The long run solutions to this set of equations is just

$$m_t = p_t + 0.99Y_t$$

$$c_t = 0.75Y_t$$

The first equation is almost a quantity theory of money. A test that the coefficient of y_t is not significantly different from 1 is a test of the quantity theory of money and can be carried out within this set-up. Note that it is difficult to complete such a test using the Engle-Granger two-step methodology. We can also test the significance of excluding certain variables from the long-term solutions. We can also re-estimate the system with these constraints imposed, if necessary. The method also allows us to test if individual variables are stationary. If the number of equations in the system is to be reduced we need to show that the coefficients of the error correction terms are not significantly different from zero in the equation corresponding to that variable. This is a pre-requisite for exogeneity. The system can also be estimated recursively to examine the stability of the long-run solutions.

If this looks to easy it is because we have oversimplified the process. The relationships we have given in this example are (over)identified. The cointegrating factors may be put in the form -

	CI1	CI2
y	0.99	-0.75
m	1	0
c	0	1
p	-1	0

The Johansen method does not directly produce such useful estimates It might produce a system such as

	CI1*	CI2*
y	1	-0.075
m	0.64	0.75
c	-0.5	1.0
p	-0.64	-0.75

In effect this system is statistically equivalent to the first system. The second set of cointegrating conditions may be derived from the fist as follows

$$CI1^* = (4\ CI1 - 3\ CI2)/6.21$$

$$CI2^* = (3\ CI1 + 4\ CI2)/4$$

A cursory examination of the second system shows how one could easily make the wrong conclusions about the coefficients of the long run relationships in the system. CI2* As written it looks like a relationship between consumption, money and prices. This is an example of the identification problem discussed in Section 2. As any two independent linear combinations of the cointegrating relationships are cointegrating relationships it is possible that one could come to a variety of wrong conclusions by examining/manipulating the estimated relationships. Valid conclusions may only be made subject to appropriate identification restrictions. In the present case we might assume that the long run relationships were

(1) a money demand equation in which the coefficient on consumption was zero

(2) a consumption function in which the coefficient on money was zero

We can derive two unique (up to a multiplicative constant) linear combinations of these two conditions. Further statistical tests must be regarded as conditional on the identification restrictions. They are not tests of the identifying restrictions. The identification restrictions must be imposed on theoretical grounds and can not be tested by statistics.

A complete implementation of this methodology is not easy but is now more accessible given the recent availability of relevant software [PCFIML (Doornik and Hendry (1994a) and "CATS IN RATS"(Hansen and Juselius (1995)].

4. NON-LINEAR MODELS

Definitions of Linearity and Non-linearity

Before we discuss non-linear models we should clarify exactly what a linear model is. Let y_t and x_t be, respectively, a scalar and an n-dimension vector valued random variable with joint probability density function $f(y_t,x_t,\theta)$ and let

$$f(y_t,x_t,\theta) = f_{y/x}(y_t/x_t,\theta_1)\, f_x(x_t,\theta_2)$$

Regression deals with inferences on θ_1. We can ignore the marginal density $f_x(x_t,\theta_2)$ if there is no relationship between θ_1 and θ_2 (exogeneity). $f_{y/x}(y_t/x_t,\theta_1)$ is the conditional density function of y_t given x_t. Regression is statistical inference based on the conditional mean $E(y_t/x_t)$ and conditional variance $V(y_t/x_t)$ derived from this conditional distribution. If second moments of x_t and y_t exist then we may write

$$Var\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} \sigma^2 & \sigma_{12}' \\ \sigma_{12} & \Sigma_{22} \end{bmatrix}$$

If $\beta_1 = \Sigma_{22}^{-1}\sigma_{12}$, $\beta_0 = E[y_t] - E[x_t]\beta_1$,

$$y_t = \beta_0 + x_t'\beta_1 + v_t$$

$$E[x_t v_t] = 0$$

then $E[y_t / x_t] = \beta_0 + E[x_t]\beta_1 + E[v_t / x_t]$

The conditional mean will not be linear if $E(v_t/x_t)$ is not zero. $\beta_0 + \mathbf{x}_t'\beta_1$ is the best linear predictor of $\mathbf{y}_t$ because β_0 and β_1 are the values of b_0 and $\mathbf{b}_1$ that minimise

$$E[(\mathbf{y}_t - b_0 + \mathbf{x}_t'\mathbf{b}_1)^2]$$

The conditional mean $E(y_t/x_t)$ is the best predictor in the sense that it minimises the mean square error and

$$E[(\mathbf{y}_t - E(y_t/x_t))^2] \le E[(\mathbf{y}_t - b_0 + \mathbf{x}_t'\mathbf{b}_1)^2]$$

When the model is linear, the conditional mean and the best linear predictor coincide. This concept of linearity is not as restrictive as might appear at first sight. When y_t and x_t are jointly normal they are linear. Again an appropriate transformation of variables may make a system linear. Log transformations are often used in economics e.g. the consumption function

$$log(C_t) = c_0 + c_1 \, log(Y_t)$$

or the quadratic inverse demand or supply function

$$p_t = \alpha_0 + \alpha_1 q + \alpha_2 q^2$$

which is linear in q and q^2 which, for estimation purposes, are separate variables. It is however possible to have functions that are intrinsically non-linear. The CES function

$$Y = A[\delta K^{-\gamma} + (1 - \delta)L^{-\gamma}]^{1/\rho}$$

is widely used as a production function in economic modelling. The theory of such systems is well developed and is surveyed in Gallant (1987) or Amemiya (1985).

We shall use the concept of white noise that is an uncorrelated random variable with constant mean and variance. Strict white noise is a sequence of independent and identically distributed random variables. If white noise is normal then it is strict white noise.

The general time series problem may be set out as follows. Suppose we have a series that is observed at say t, t-1, t-2,...... A model for such a series is a relationship of the form[13]

$$h(X_t, X_{t-1}, X_{t-2} ..) = \varepsilon_t$$

where ε_t is a strict white noise and h is some prescribed function. If the model is invertible we can write X_t as a function of $\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, ...$ If this program has been completed we have found the best possible model for X_t. In general, the range of functions h available must be restricted. If h is linear we may write

$$\sum_{u=0}^{\infty} h_u X_{t-u} = \varepsilon_t$$

and if this is invertible

$$X_t = \sum_{u=0}^{\infty} g_u \varepsilon_{t-u}$$

This is our basic linear time-series model. We have expressed X_t as a linear combination of present and past innovations. All stationary/invertible AR, MA ARMA models may be put in this form. Consider now the case where h is non-linear. The following simple example illustrates some of the consequences. In inverted form let

$$y_t = \varepsilon_t + \beta\varepsilon_{t-1}\varepsilon_{t-2} \quad \varepsilon_t \text{ is iid } N(0,\sigma^2)$$
$$E[y_t] = 0 \qquad Var[y_t] = \sigma^2(1+\beta^2\sigma^2)$$
$$\gamma(\tau) = E[y_t y_{t-\tau}] = 0$$

Conditional Mean is $E[y_t / x_t] = \beta\varepsilon_{t-1}\varepsilon_{t-2} \quad MSE = \sigma^2$

Best Linear Estimate is 0 with variance $\sigma^2(1+\beta^2\sigma^2)$

Thus, the Best Linear Estimate and the conditional mean do not coincide. Again we regard a model as non-linear when it has this property.

Advantages and Disadvantages of Linear and Non-Linear Models

Linear models have certain advantages:

1. In many cases a linear model will provide a good local approximation to a non-linear problem.

2. In economics, in particular, we have relatively short data series that are subject to considerable measurement error or which may be

proxies for the true variables. Such data may not support a more elaborate non-linear model.

3. Mathematically a complete theory is available for linear models. Closed expressions may be derived for means, variances, forecast values, equilibrium etc. In non-linear theory such closed solutions will, in general, not exist and we may have to depend on numerical solutions or simulation exercises to derive numerical solutions to our problems.

4. Computation time is generally smaller and a wider range of packages are available for linear models.

At the same time linear models have certain limitations.

I. The deterministic part of a linear difference equation imposes limitations on the long-run behaviour of a stochastic linear model. If a linear model is stable it will converge to a fixed point and this fixed point will not depend on initial conditions. In the presence of unit roots the point of convergence will depend on initial conditions. Otherwise an unstable model will diverge to infinity. Long-term oscillations can only occur for pathological values of the parameters in a linear model. For more details see, for example, chapters 2 and 11 of Lütkepohl (1991).

II. They can not be used to model strong asymmetric behaviour.

III. They can not be used to model strong bursts of activity.

IV. They can not model time irreversibility.

Various aspects of economic theory that lead to linear and non-linear dynamic models are described in Frain (1992). We now look at various kinds of non-linear models that are or may be of use in econometrics

Bilinear Models

Returning to our general non-linear model we may use Taylor's theorem on the inverted expression to get

$$X_t = \mu + \sum_{u=0}^{\infty} g_u \varepsilon_{t-u} + \sum_{u=0}^{\infty}\sum_{v=0}^{\infty} g_{uv}\varepsilon_{t-u}\varepsilon_{t-v} + \sum_{u=0}^{\infty}\sum_{v=0}^{\infty}\sum_{w=0}^{\infty} g_{uvw}\varepsilon_{t-u}\varepsilon_{t-v}\varepsilon_{t-w} + \ldots.$$

This is known as a Volterra series. If $g_{uv} = g_{uvw} = \ldots = 0$ the expression reduces to an AR process. We can now, by analogy with the kind of arguments underlying the ARMA model, introduce the Bilinear process :

$$X_t = \sum_{j=0}^{\infty} a_j X_{t-j} + \sum_{j=0}^{\infty} c_j \varepsilon_{t-j} + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} g_{ij} X_{t-i}\varepsilon_{t-j}$$
$$c_0 = 1 \ \ and \ \ \varepsilon_t \ \ is \ \ a \ \ strict \ \ white \ \ noise$$

The three graphs below show simulations of the following series:

$$A : X_t = 0.4X_{t-1} + 0.4X_{t-1}\varepsilon_{t-1} + \varepsilon_t$$
$$B : X_t = 0.4X_{t-1} + 0.8X_{t-1}\varepsilon_{t-1} + \varepsilon_t$$
$$C : X_t = 0.8X_{t-1} - 0.4X_{t-2} + 0.6X_{t-1}\varepsilon_{t-1} + 0.7X_{t-2}\varepsilon_{t-1} + \varepsilon_t$$

The ε_t generating the series consist of 1000 observations of an iid N (0,1). variable and the same set are used to generate each series

The graph of the first series looks somewhat similar to that of a standard AR process. The second series has a stronger Bilinear element and the series shows bursts of activity. In the third series the Bilinear element is very strong. The series has three bursts of volatility that drown out any variation in the remainder of the series. The theoretical basis of the Bilinear process is that by an appropriate choice of parameters a bilinear model can approximate any "well behaved" Volterra series relationship over a finite period of time (Brockett (1976))

Simulation of series A

Simulation of series B

Simulation of Series C

Threshold Autoregressive (TAR) Models

A certain body of econometric theory has been devoted to disequilibrium models and sets out a basis for the analysis of regimes that are subject to change. Switching regressions and similar methods are used to model systems which switch between positions of excess supply, excess demand and equilibrium. We apply similar methods to models with self-selectivity (i.e. individuals choosing between professions, evaluation of programs where the individual has a choice of participation/non-participation). Maddala (1983) surveys this type of analysis.

In time series analysis the analogue of a switching regression is the TAR model. Tong (1990) describes the theory of these models. The regimes in a TAR model and the rules regarding switching can be defined in several ways. In the simplest version, the regimes are defined in terms of a lagged value of an exogenous variable, i.e.

$$\begin{aligned} R_1 &: a_1 \le X_{t-d} \le b_1 \\ R_2 &: a_2 \le X_{t-d} \le b_2 \\ &\cdots\cdots\cdots\cdots \\ R_n &: a_n \le X_{t-d} \le b_n \end{aligned}$$

The n regimes are of course disjoint and d is a delay parameter, which may be zero. The endogenous variable then follows a different AR process in each regime. There are, also, multivariate input/output variations of this system.

There are various extensions to the TAR family of systems. The Exponential Autoregressive model (EAR) in its simplest form has two regimes. At each observation it has a probability of p of following one process and of (1-p) of following the other. In the Markov Model the transition from regime to regime is governed by a Markov process. We can even generate "chaos" (Fractals) with a deterministic (?) TAR. Here $X_t = a_1^{(J_t)} X_{t-1} + a_0^{(J_t)}$ where J_t is a sequence of iid random variables independent of X_t.

The smooth TAR is a further development of this process. Here X_t is a mixture of two AR processes.

$$X_t = a_0 + a_1 X_{t-1} + (b_0 + b_1 X_{t-1}) F(y_{t-d}) + \varepsilon_t$$

where $F(y_{t-d})$ is a transition function that takes values between zero and one. Two forms in particular have been used as transition functions.

Logistic transition function

$$F(y_{t-d}) = (1+\exp[-\gamma(y_{t-d}-c)])^{-1}, \ \gamma > 0$$

Exponential transition function

$$F(y_{t-d}) = 1-\exp[-\gamma(y_{t-d}-c)^2], \ \gamma > 0$$

The transition function changes monotonically and smoothly with y_{t-d}. In the Logistic STAR the model tends to an AR process as $\gamma \to 0$ and to a TAR as $\gamma \to \infty$. The contraction and expansion phases of the cycle may have different dynamics. In the Exponential STAR as $\gamma \to 0$ or ∞ the model becomes linear. The dynamics in the middle ground may differ from those at the extremes. There have been several applications of TAR models in economics. The following four are mentioned as typical examples:

Teräsvirta and Anderson (1992) - apply Smooth TAR models to production indices for 13 countries and Europe. Tests rejected linearity and estimated Star models indicated that the non-linearity is needed mainly to describe the response to large negative shocks.

Teräsvirta (1994) - ESTAR applied to US per capita GNP

Hansen (1992b) - estimated an EAR model of GNP that fitted
better than an AR (4) or the Markov TAR of Hamilton (Econometrica 1989)

Town (1992) - estimated a Markov switching model of Acquisition Waves.

ARCH Models

Engle (1982) proposed a model of the conditional variance, ARCH, as opposed to those of the conditional mean. Bollerslev, Chou and Kroner (1992), have recently, surveyed the theory and application of ARCH models and their derivatives.

By way of introduction consider the basis AR (1) model.

$$y_t = \phi y_{t-1} + \varepsilon_t, \varepsilon_t \text{ is iid } N(0,\sigma^2), |\phi| < 1$$
$$E[y_t] = 0, \quad Var[y_t] = \sigma^2(1-\phi^2)$$

The conditional mean and conditional variance are given by.

$$E[y_t / y_{t-1}] = \phi y_{t-1}, \quad Var[y_t / y_{t-1}] = \sigma^2$$

Here the Conditional Variance is independent of the history of the process. A similar result may be proven for ARMA models and for Linear regressions with weakly exogenous variables. The ARCH family of models allows us to relax this restriction on the conditional variance. In an ARCH model the conditional distribution of X_t given I_t (Lags X_t and values of the exogenous variables) is given by

$$N(g_t, h_t) \text{ where } g_t = Z'\beta \text{ and } h_t = \alpha_0 + \sum_{i=0}^{q} \alpha_i \varepsilon_{t-1}^2, \quad \varepsilon_t = y_t - g_t$$

In this case the conditional variance is modelled as an AR process that gives rise to the description Autoregressive Conditional Heteroscedastic or ARCH.

We may illustrate some of the properties of the ARCH family by the following example of an AR(1) process with ARCH(1) errors.

$$y_t = \phi y_{t-1} + \varepsilon_t$$
$$E[\varepsilon_t / \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots] = 0$$
$$Var[\varepsilon_t / \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots] = h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$$
$$|\phi| < 1, \quad h_t > 0, \quad \alpha_0 > 0, \quad \alpha_1 \geq 0$$
$$Var[\varepsilon_t] = \alpha_0 / (1-\alpha_1) = \sigma^2$$
$$h_t - \sigma^2 = \alpha_1(\varepsilon_{t-1}^2 - \sigma^2)$$

Note the following points:

- The errors are uncorrelated but not independent.
- The tails of the distribution are fatter than those of a normal distribution. .
- The conditional variance is greater than the unconditional variance when $\varepsilon_{t-1}^2 > \sigma^2$. Thus, large values of the conditional variance are expected to follow large values of the variance

The ARCH concept has been greatly extended and we can now refer to an ARCH family. Some of the more important members of that family are -

- ARMACH (GARCH) where the conditional variance is an ARMA process.
- ARCH-M where the volatility effects the expected mean.
- IGARCH where the conditional variance follows an integrated process.

The ARCH family of models have been extensively used in Economics and in particular in Finance where the rate of return is related to risk which is measured by volatility or variance. Bollerslev et al. (1992) give more than 300 references to recent work in areas such as stock return data, modelling of interest rates and foreign exchange rates. Fell (1994a and 1994b) uses ARCH to model volatility in the foreign exchange market and to examine the effects of trading rules on that market.

In this section I have only provided a very basic introduction to the theory and applications of non-linear systems. With the greater availability and reduced cost of computer systems it has become feasible to apply these to a wide range of problems with some worthwhile results. Further improvement in theory and facilities will make these procedures available to an even wider range of problems. I have not mentioned problems of testing or estimation of non-linear systems. The books by Priestly(1989), Tong(1990) and Granger and Teräsvirta(1993) or the references quoted in this section give greater detail.

Chaos

The linear model is often described as simple by way of contrast to the complex behaviour of biological and similar systems. The ideas prevalent in the 1960's suggested that observed complex phenomena such as turbulent behaviour in fluids could only be described by an extremely large number of equations. The American meteorologist Edward Lorenz (1963) was the first to discover how a dynamic system involving only a few variables could model such phenomena. His work did not receive any attention for nearly ten years. To-day we known that a simple non-linear model may be able to model many such complex systems. Frain (1992) reviews these types of processes and their implications for economic theory.

Perhaps the easiest way to demonstrate the concept of chaos is by way of a simple univariate example. The logistic system is one such system.

$$X_{t+1} = \lambda X_t(1-X_t) \qquad 0 \leq X_t \leq 1 \qquad 0 \leq \lambda \leq 4$$

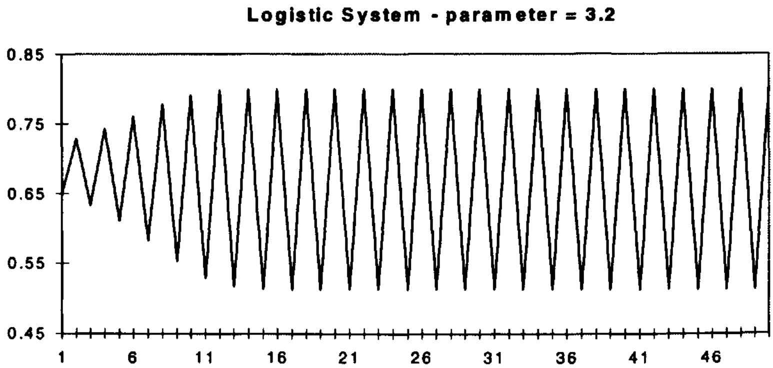
The equilibrium points of this system are

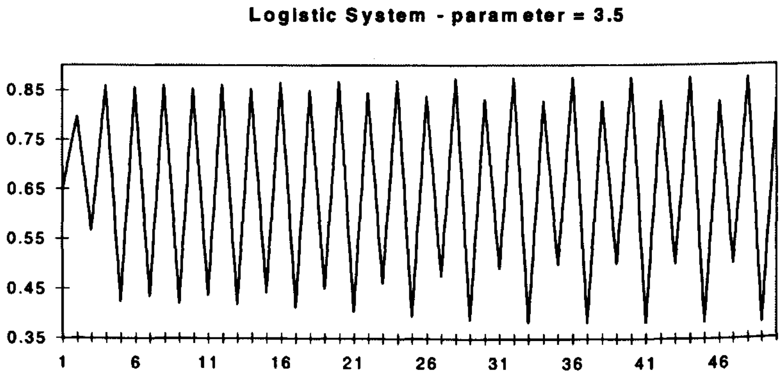
$$X = \lambda X(1-X) \qquad \text{or } X = 0 \text{ or } 1-1/\lambda$$

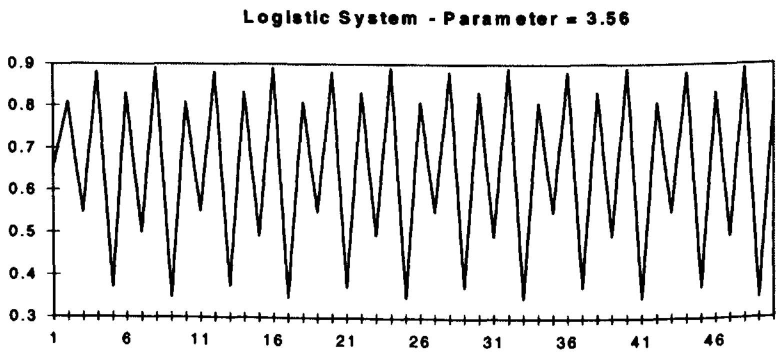
Examine the behaviour of this system and in particular its relationship to the non-zero equilibrium point as λ increases. If $0 \leq \lambda \leq 1$ this root is negative and can not be reached. For $1 \leq \lambda \leq 3$ the root is stable. For $\lambda > 3$ the equilibrium is unstable. As λ is increased above 3 at first a limit cycle of length 2 appears, then one of 4, then of 8 then ....then of 2^n .... till we arrive at a value of about 3.57 where the frequency of the cycle is infinite and the series behaves as a random stochastic process. As we increase λ further more cycles appear.

The graphs below illustrate these points. With $\lambda = 3.2$ the system is generating limit cycles of period 2. At 3.5 the limit cycles are of length 4. A close examination of the graph at 3.56 shows limit cycles to period 8. At 3.739 and 3.853 we have limit cycles of 5 and 3 respectively. When $\lambda = 4$ we have chaos. If the sequence achieves a particular number that number will never be repeated again. The sequence, although completely deterministic, has many of the properties of a random stochastic process and hence the term chaos.

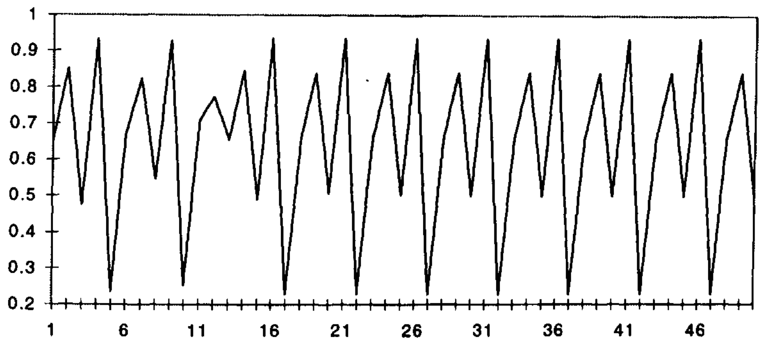
The next two graphs show the sensitivity of the logistic system to starting values. On the upper graph we show two series with initial values of 4.0 and 4.0000001. The lower half shows the differences of these two series. The two series follow one another very closely for the first 20 observations and then diverge. This clearly indicates the sensitivity of chaotic systems to initial conditions. - the butterfly effect.


Logistic System - parameter = 3.2
0.85
0.75
0.65
0.55
0.45
1
6
11
16
21
26
31
36
41
46

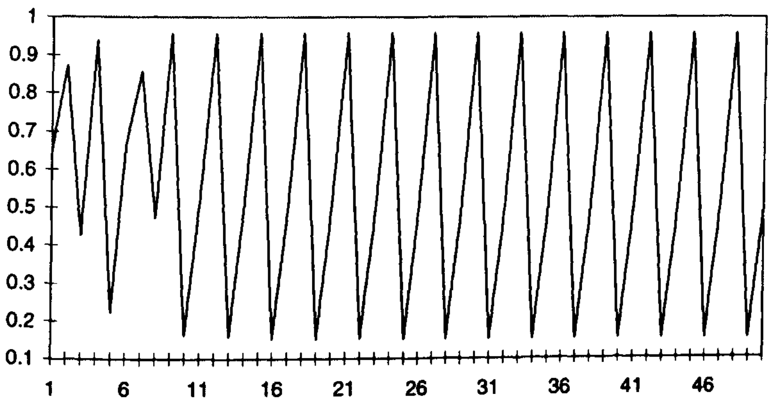

Logistic System - parameter = 3.5
0.85
0.75
0.65
0.55
0.45
0.35
1
6
11
16
21
26
31
36
41
46


Logistic System - Parameter = 3.56
0.9
0.8
0.7
0.6
0.5
0.4
0.3
1
6
11
16
21
26
31
36
41
46

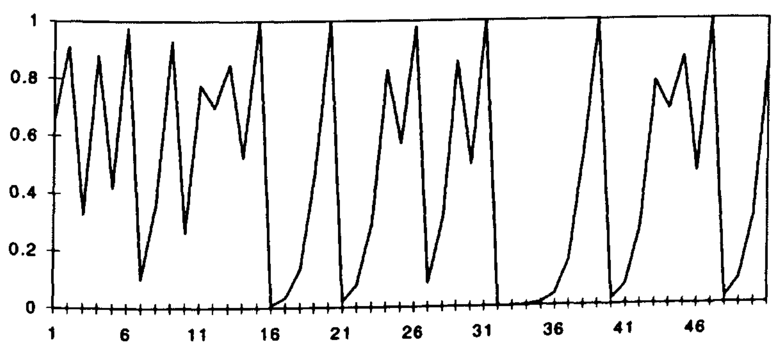
Logistic System - Parameter = 3.739



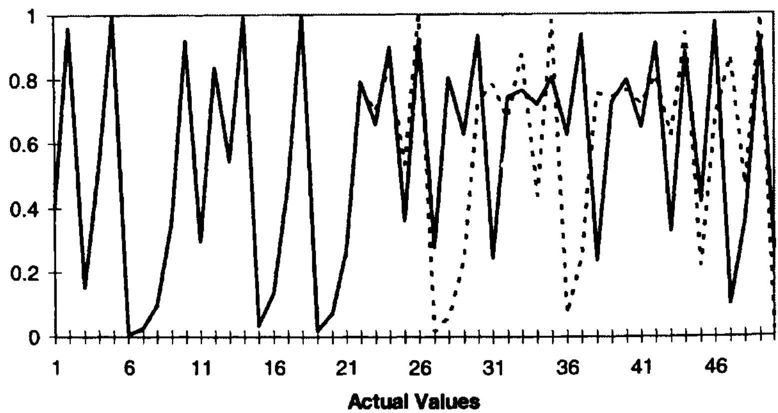
Logistic System - Parameter = 3.835



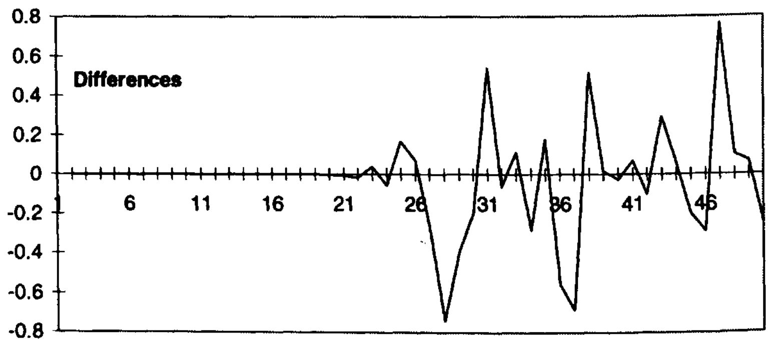
Logistic System Parameter = 4.0



Dependence of Chaotic System on starting Values



Dependence of Chaotic System on Starting Values



It is generally accepted that many economic systems are non-linear. However, we do not know if that non-linearity is sufficient to generate chaos. Day(1993) shows how chaos can arise in a simple growth model. Day and Shafer (1985) demonstrate chaos in a Keynesian macromodel. Grandmont sets up an intergenerational model of the business cycle that is not very different from those of the equilibrium business cycle theorists. He proves that the solution to the model is chaotic. De Grauwe and Dewatcher (1990) and De Grauwe and Vansanten (1990) build chaotic models of the exchange rate markets. The cause of chaos in this model is the interaction of chartists and fundamentalists operating on the market.

Brock (1986) concludes that in order to get complex dynamics and instability in economics we must do at least one of the following:

1) introduce agents that discount the future heavily;
2) abandon concavity assumptions on tastes and technology; increasing returns along with externalities might lead to examples of complex dynamics - growth and decay of cities;
3) abandon the assumption of complete markets;
4) abandon the assumption of price taking agents;
5) impose complex preferences on technology;
6) abandon assumption that system is in equilibrium;
7) allow interactions between the actions of agents and the preferences of others or
8) introduce exogenous "forcing" agents.

Showing that chaos might occur in theory does not mean that it occurs in practice. Statistical research has shown that non-linearity occurs but the evidence for chaos is at best indecisive.

5. MODEL SPECIFICATION

"The race is not always to the swift nor the battle to the strong, but that's the way to bet".

The previous three sections of this paper deal largely with the problem of estimating a known model with data measuring the actual variables in the model. In real life, we are not as fortunate and encounter many problems.

1. We do not have a complete specification of the model or we have alternative specifications.
2. The data are only proxies for the variables in the model and may be subject to error.

Econometric modelling is like playing a musical instrument. To play it really well a large amount of technique is necessary but not sufficient. One needs a considerable amount of feeling for the music. This does not stop the rest of us amusing ourselves by playing the piano or violin or some other instrument but most of us would not pretend that we have any particular talent in that line and would not impose our playing on others.

The problems set out in this section are not covered in the main text books on econometrics. Granger (1990) contains a good collection of articles covering what might be called the LSE or Hendry approach; the VAR or Sims approach; and the Bayesian or Leamer approach and the macroeconometric approach[14]. The attribution of the approaches to Hendry, Sims and Leamer or to LSE follows that of Pagan (1987) who provides greater detail of each procedure and is not intended to take from the large number of others who have contributed to their development.

The three methodologies provide contrasting advice on econometric procedures. The Bayesian has been least used because

1. It requires the assumption of prior knowledge and Bayesians appear to have problems agreeing priors and
2. There is a lack of a good range of computer software to implement the procedures and
3. The procedures require more powerful computer facilities than the alternatives.

The arrival of cheaper and more powerful computer facilities has greatly reduced the third problem. This will provide an incentive for better software that may in the long run lead to more agreement on appropriate priors.

LSE or Hendry Approach

The statistical foundations of the LSE approach are set out in Spanos (1986), Hendry (1995) and Doornik and Hendry (1994 a,b). This approach emphasises that econometric models are models of observed data sets. The model must take account of the economic process and the measurement process. The result of this amalgamation is known as the Data Generating Process.

We start by constructing a general model. We then aim to reduce this general model to a parsimonious model that is a valid representation of the data generating process and performs at least as well as other models. The general model may be a multivariate VAR possibly with non-modelled variables added. The minimum required for the general system is that

1. The error term is a constant variance uncorrelated zero mean (vector) system.
2. Any unmodelled variables are exogenous for the parameters being estimated.
3. The parameters being estimated are constant.

The integration and cointegration states of the variables are determined using the univariate or multivariate methodology described in section 3. Subsequent inference is dependent on the results of this analysis. Economic theory and statistical analysis are used to eliminate certain variables/lags from the model.

The exogeneity of certain variables can be tested indirectly using tests of parameter constancy. Intuitively, any estimated parameters cannot change when an exogenous variable is changed. If we wish to treat as exogenous a variable in a VAR with cointegrating constraints the error correction terms must have zero coefficients in the equation in the VAR defining that variable. It would be prudent in many cases if this was tested before undertaking the estimation of a reduced dimension system or a single equation.

The simplified system must be well behaved. PC Give Ver 8 [Doornik and Hendry 1994b] produces the following statistics, diagnostics etc. for an OLS regression.

1. Correlations	sample means variances and correlations of the variables in the regression.
2. Estimation Equation	
3. Standard Errors	

4. HCSE — Heteroscedastic - consistent standard errors. Large differences between 3 and 4 indicated problems with constancy of variance.
5. t-values
6. Squared partial correlations
7. Parameter instability statistics — Hansen (1992) - valid only in the absence of non-stationary variables
8. R^2
9. Equation standard error
10. F-statistic
11. Durbin Watson
12. Residual Sum of Squares
13. Parameter instability tests
14. Information
15. Seasonal means of differences
16. R^2 relative to differences and seasonals
17. Variance - covariance matrix

If, observations are withheld for forecasting purposes.

18. Analysis of 1-step forecasts
19. X^2 forecast test on 1-step forecasts
20. Chow tests
21. t test for zero forecast innovation mean

This is followed by a graphical analysis which includes -

1. Actual and Fitted values
2. Cross-plot of Actual and Fitted Values
3. Scaled Residuals
4. Forecasts and Outcome
5. Residual Correlation
6. Residual Spectrum
7. Residual Density
8. Residual Distribution

Recursive methods estimate the model from the start of the data up to each point at which the model can be estimated. There are options to analyse the results of recursive estimation but it is recommended that this be completed graphically. These statistics are concerned with the constancy of model. The following are produced:

1. Coefficients ± 2 standard errors
2. t- values for coefficients
3. Residual Sum of Squares for each regression
4. Standardised innovations
5. 1-Step residuals ± 2
6. 1-Step Chow tests
7. Break point F-tests
8. Forecast F-tests

The lag structures are then analysed and roots and coefficient sums of the polynomials given.

Diagnostic tests include residual autocorrelation, conditional heteroscedasticity, normality, unconditional heteroscedasticity/functional form and omitted variables.

Finally, the performance of the simplified model is compared to that of the general model. The simplified model must be able to explain the system at least as well as the general model. Procedures are also given to test its efficiency against that of previous models. If you have got through that list you might understand Hendry's dictum. Test! Test! Test!

Hendry (1987) proposes four 'golden prescriptions' of econometrics.

I. *think brilliantly: if you think of the right answer before modelling, then the empirical results will be optimal and, of course, confirm your brilliance. Many conventional textbooks simply assume that the model s correct - we will not do so below, although the methods proposed deliver the right results if this case happens to apply.*

II. *be infinitely creative: if you do not think of the correct model before commencing, the next best is to think of it as you proceed. While no valid constructive method can be proposed, data evidence can help guide model development in a systematic manner.*

III. *be outstandingly lucky: if you do not think of the 'true model' before starting nor discover it en route, then luckily stumbling over it before completing the study is the final sufficient condition. This may be the most practical of these suggestions.*

Failing this last prescription:

IV. *stick to doing theory.*

Hendry (1995) comments on the prescriptions

> *"These sufficient conditions are tantamount to the assumption of omniscience of the modeller and we can not rely on their sustaining a viable methodology. Fortunately, these prescriptions are not necessary. The book argues that no realistic sufficient conditions can be established which ensure the discovery of a 'good' empirical model, nor are any required for empirical econometrics to progress. However, there are a number of necessary conditions which can rule out many poor models, allowing us to focus on the best remaining candidates".*

The influence of the LSE approach is considerable. Hendry and Wallis (1984) contains a useful collection of applications.

The VAR or Sims Approach

Sims (1980) argued that many of the assumptions made to identify standard econometric models were wrong. He showed that models with useful descriptive characteristics could be built using a VAR methodology. The models could also be used to carry out tests of economically meaningful hypotheses. Sims (1980) builds VAR models for the US and Germany based on money GNP (y), unemployment rate (w), price level (p) and an import price index (x). All variables are regarded as endogenous (i.e. each variable is on the right-hand side of one equation). Each variable is regarded as a function of its own lags and the lags of the other variables, for example,

$$
\begin{aligned}
m_t = \mu_1 &+ \alpha_{11} m_{t-1} + \alpha_{12} m_{t-2} + \alpha_{13} m_{t-3} + \alpha_{14} m_{t-4} \\
&+ \alpha_{21} y_{t-1} + \alpha_{22} y_{t-2} + \alpha_{23} y_{t-3} + \alpha_{24} y_{t-4} \\
&+ \alpha_{31} u_{t-1} + \alpha_{32} u_{t-2} + \alpha_{33} u_{t-3} + \alpha_{34} u_{t-4} \\
&+ \alpha_{41} p_{t-1} + \alpha_{42} p_{t-2} + \alpha_{43} p_{t-3} + \alpha_{44} p_{t-4} \\
&+ \alpha_{51} x_{t-1} + \alpha_{52} x_{t-2} + \alpha_{53} x_{t-3} + \alpha_{54} x_{t-4} + \varepsilon_{1t}
\end{aligned}
$$

for a VAR of order 4 (4 lags). There are 4 similar equations for the other variables. Under certain conditions each equation can be estimated by OLS and we can use likelihood ratio tests to test hypotheses such as blocks of coefficients being insignificant (e.g. causality tests).

Given certain conditions such a system will tend to a stable equilibrium. The behaviour of such a system can be examined by estimating the effect of a unit (variance) shock to one variable in one time period on itself and the other variables in the model. The results of this impulse analyses are generally presented in graphical form.

Quite often it will be found that there is correlation between the errors in different equations. In such cases the impulse analysis described above is not sufficient. It is always possible to arrange the variables as follows. Let x_1 x_2 x_3 and x_4 be a rearrangement of the variables in the VAR such that:

1) the equation for x_1 contains only lags

2) the equation for x_2 contains contemporaneous x_1 and lags

3) the equation for x_3 contain contemporaneous x_1 and x_2 and lags

4) the equation for x_4 contains contemporaneous x_1 x_2 x_3 and lags

and the error terms are uncorrelated. In such a case the impulses are said to be orthogonal. The system implies that x_1 is exogenous for all the other variables, x_1 and x_2 are exogenous for x_3 and x_4 and x_1 x_2 and x_3 are exogenous for x_4. The ordering of the variables is in general dictated by economic theory. Mathematically there is no unique way of carrying out this ordering of variables and even with a particular ordering there is a multitude of ways of setting up the system. The results of the analysis are also critically dependent on the ordering used.

Given a model with orthogonal innovations further analysis is possible. The forecast error variance of each variable at various horizons may be decomposed into components due to innovations in the other variables. Again the results are dependent on the ordering chosen. This analysis is known as a Forecast Error Decomposition.

This type of analysis can be extended to take account of non-stationary and cointegrated variables. Note that in such cases impulse response functions will show persistent effects. The problems of interpretation that apply to the stationary case also apply to the non-stationary case. For a detailed account see Lütkepohl (1991).

A further development in VAR analysis is the SVAR or structural VAR. For details see Giannini (1992). The following example is due to Pagan (1995). Let y_t be output, m_t real money and i_t an interest rate. An orthogonalisation based on an ordering m_t i_t y_t would be as follows:

Hendry (1995) comments on the prescriptions

> *"These sufficient conditions are tantamount to the assumption of omniscience of the modeller and we can not rely on their sustaining a viable methodology. Fortunately, these prescriptions are not necessary. The book argues that no realistic sufficient conditions can be established which ensure the discovery of a 'good' empirical model, nor are any required for empirical econometrics to progress. However, there are a number of necessary conditions which can rule out many poor models, allowing us to focus on the best remaining candidates".*

The influence of the LSE approach is considerable. Hendry and Wallis (1984) contains a useful collection of applications.

The VAR or Sims Approach

Sims (1980) argued that many of the assumptions made to identify standard econometric models were wrong. He showed that models with useful descriptive characteristics could be built using a VAR methodology. The models could also be used to carry out tests of economically meaningful hypotheses. Sims (1980) builds VAR models for the US and Germany based on money GNP (y), unemployment rate (w), price level (p) and an import price index (x). All variables are regarded as endogenous (i.e. each variable is on the right-hand side of one equation). Each variable is regarded as a function of its own lags and the lags of the other variables, for example,

$$
\begin{aligned}
m_t = \mu_1 &+ \alpha_{11} m_{t-1} + \alpha_{12} m_{t-2} + \alpha_{13} m_{t-3} + \alpha_{14} m_{t-4} \\
&+ \alpha_{21} y_{t-1} + \alpha_{22} y_{t-2} + \alpha_{23} y_{t-3} + \alpha_{24} y_{t-4} \\
&+ \alpha_{31} u_{t-1} + \alpha_{32} u_{t-2} + \alpha_{33} u_{t-3} + \alpha_{34} u_{t-4} \\
&+ \alpha_{41} p_{t-1} + \alpha_{42} p_{t-2} + \alpha_{43} p_{t-3} + \alpha_{44} p_{t-4} \\
&+ \alpha_{51} x_{t-1} + \alpha_{52} x_{t-2} + \alpha_{53} x_{t-3} + \alpha_{54} x_{t-4} + \varepsilon_{1t}
\end{aligned}
$$

for a VAR of order 4 (4 lags). There are 4 similar equations for the other variables. Under certain conditions each equation can be estimated by OLS and we can use likelihood ratio tests to test hypotheses such as blocks of coefficients being insignificant (e.g. causality tests).

Given certain conditions such a system will tend to a stable equilibrium. The behaviour of such a system can be examined by estimating the effect of a unit (variance) shock to one variable in one time period on itself and the other variables in the model. The results of this impulse analyses are generally presented in graphical form.

Quite often it will be found that there is correlation between the errors in different equations. In such cases the impulse analysis described above is not sufficient. It is always possible to arrange the variables as follows. Let x_1 x_2 x_3 and x_4 be a rearrangement of the variables in the VAR such that:

1) the equation for x_1 contains only lags

2) the equation for x_2 contains contemporaneous x_1 and lags

3) the equation for x_3 contain contemporaneous x_1 and x_2 and lags

4) the equation for x_4 contains contemporaneous x_1 x_2 x_3 and lags

and the error terms are uncorrelated. In such a case the impulses are said to be orthogonal. The system implies that x_1 is exogenous for all the other variables, x_1 and x_2 are exogenous for x_3 and x_4 and x_1 x_2 and x_3 are exogenous for x_4. The ordering of the variables is in general dictated by economic theory. Mathematically there is no unique way of carrying out this ordering of variables and even with a particular ordering there is a multitude of ways of setting up the system. The results of the analysis are also critically dependent on the ordering used.

Given a model with orthogonal innovations further analysis is possible. The forecast error variance of each variable at various horizons may be decomposed into components due to innovations in the other variables. Again the results are dependent on the ordering chosen. This analysis is known as a Forecast Error Decomposition.

This type of analysis can be extended to take account of non-stationary and cointegrated variables. Note that in such cases impulse response functions will show persistent effects. The problems of interpretation that apply to the stationary case also apply to the non-stationary case. For a detailed account see Lütkepohl (1991).

A further development in VAR analysis is the SVAR or structural VAR. For details see Giannini (1992). The following example is due to Pagan (1995). Let y_t be output, m_t real money and i_t an interest rate. An orthogonalisation based on an ordering m_t i_t y_t would be as follows:

$$m_t = \text{lags} + \text{error}$$

$$i_t + a_1 m_t = \text{lags} + \text{error}$$

$$y_t + a_2\, i_t + a_3\, m_t = \text{lags} + \text{error}$$

In addition to the lag coefficients there are three parameters to be estimated. Suppose however we postulate an IS-LM system as follows:

$y_t - b_1\, i_t$	$= \text{lags} + \text{error}$	IS
$i_t - b_2 y_t - b_3\, m_t$	$= \text{lags} + \text{error}$	LM
m_t	$= \text{lags} + \text{error}$	money supply

which possess the same number of coefficients as the earlier triangular form. Thus, the systems are observationally equivalent. The impulse response functions and resulting policy analyses are not the same.

An alternative way of fixing coefficients is to place restrictions on the impulse response functions.

Bayesian or Leamer Approach

The extreme bounds methodology is discussed and examined in detail in Leamer (1983), McAleer et al (1985), Breusch (1985), Cooley and LeRoy (1986) and Leamer (1985). Pagan (1987) reduces Leamer's method to four steps:

1) *Formulate a general family of models.*

2) *Decide what inferences are of interest and express these in terms of the parameters and form 'tentative' prior distributions that summarise the information not contained in the given dataset.*

3) *Consider the sensitivity of inferences to a particular choice of prior distributions. Sometimes step (3) terminates the process, but when it appears that inferences are sensitive to the prior specification this step is only a warm up for the next one.*

4) *Try to obtain a narrower range for the inferences. If the restrictions involved in this latter case are too 'implausible' one concludes that the inference based on the data is fragile.*

This is the extreme bounds analysis of Leamer (1983). This sensitivity of results to the variables included is an important topic to which far too little attention has been paid. It is often the case that a minor change in the model being estimated can reverse the sign of an estimated coefficient.

One can also do a form of Extreme Bounds analysis without using Bayesian theory. Frain and O'Connell (1989) in an analysis of the effect of exchange rate changes on inflation estimate 300 regressions to test the sensitivity of their analysis to model specification and find that their model is robust and find that their conclusions hold across all the specifications considered. This kind of analysis is as important as the battery of specification and miss-specification tests that are used in applied work. Any result that has not got a certain degree of robustness can not be of great value.

Macroeconometric Modelling

I shall now outline how the methods and philosophy already described are being applied in the Bank to the building of a small macroeconometric model. Our intention is that this model be kept small and that the model databank will be based directly on CSO data, the IFS databank and other data readily available in the Bank. Great importance will be attached to ease of maintenance of the model and updating of databanks. It is hoped that the model will

- provide a useful data summary
- be a platform for more specialised analyses which it may be possible to estimate/simulate as extended subsectors of the model
- help to establish consistency of forecasts
- assist in the accumulation and consolidation of economic knowledge
- form part of the LINK system of national models.

The first part of the process is to estimate the main equations in the model e.g. consumption, investment, import and export functions. These will be estimated using the LSE general to specific approach. At first, equations will be estimated using OLS but we plan at an early stage to divide our variables into modelled and non-modelled. At this stage we plan to repeat our OLS estimations using partial information estimation procedures. Each individual equation will be subjected to a large number of tests. When the system is completed we will evaluate it in terms of fit, forecasting ability, multipliers etc. At that stage some amendments may be necessary and they can be made again using partial information methods. When the model appears to be working well we will attempt to estimate it again using full information methods. This final version of the model will be tested and any problems arising examined. We hope that this process will produce a useful model of the Irish economy. Some initial work on the model is in Frain, Howlett and McGuire(1995).

6. COMPUTERS IN ECONOMETRICS

The procedures described in the previous four sections are complicated and require considerable computer resources. Ten years ago these resources would have been expensive. To-day the availability of cheap computer power has made it possible to complete them at very little cost. The first generation of econometricians would have found it impossible to do anything but the simplest of calculations as they worked entirely by hand. Even after the advent of mechanical calculators, econometric work required inputs that today sound unbelievable. It is interesting to recall the following extract from a 1946 report of the Cowles Commission as quoted in Berndt (1991):

> *"In answer to a question by Girshick, Koopmans mentioned that one supervisor and one or two computers had worked two to three months on an eight equation system, by hand calculating. Tubin estimated that a ten equation system with fifty unknown parameters by hand calculating with an ordinary calculator required seventy 24-hour computer days".*

The advent of computers did not altogether relieve this situation. Longley (1967) appraised various *"least-squares programs for the electronic computer from the point of view of the user"*. He compared the results of eight regressions on six independent variables using a variety of programs on six different mainframe computers. With identical inputs all except four programs produced outputs which differed from each other in every digit. The cause of the problems was the use of poor algorithms rather than limited hardware. The algorithms were based on the methods used for hand calculators that carried more significant digits. The Longley analysis should provide a timely reminder of the problems of numerical instability that can lead to even greater problems in more elaborate analyses.

The mainframe computer was expensive, not very easy to use and often required a considerable number of support staff. It was not until the arrival of the microcomputer that much econometric analysis became feasible. In Frain (1987) I examined a number of econometric programs that ran on a basic IBM PC (8086) with a math-coprocesser. The programs included

YSTAT ver 1.2

PC-GIVE ver 4.2

SHAZAM ver 4.2 and 5.1

RATS ver 2.01 and

GAUSS ver 1.49 (b)

While the hardware imposed certain constraints it was clear that it was possible to complete a great deal of real work. Each of the individual programs had its own strengths. In some cases it was much easier to work with them than with standard mainframe packages. Only analyses that were very elaborate or required a large dataset required a mainframe.

To-day most of the hardware constraints have been removed and software has been improved. It is now possible to complete most[15] analyses on PC's. The low cost makes it possible to complete many analyses that might have been too expensive on mainframes. Reduced costs and improved computer facilities have lead to improvements in software and the cycle of reducing costs and improved facilities is likely to continue.

Thus, econometric analysis is becoming increasingly complicated and demands considerable knowledge of mathematics, statistical theory, numerical methods and computer science in addition to a deep understanding of the economy. In many ways the packages are becoming more user friendly and can give the impression that their use is simply a matter of pushing an appropriate button and displaying results on a screen. In the Bank we now use a variety of econometric packages

MICROFIT	Ver 3.21
PCGIVE	Ver 8.00
REGX	Ver 92.6
RATS	Ver 3.10c
SHAZAM	Ver 7.0
GAUSS	Ver 3.24
MATHEMATICA	Ver 2.23

MICROFIT, PCGIVE and REGX are, in the first instance menu-driven but each has its own way of extending the set procedures with forms of batch or matrix manipulation languages.

MICROFIT is probably the easiest of all to use. It offers a range of single equation estimation methods.

Ordinary Least Squares	
IV (2SLS)	Instrumental Variables
AR (J)	Autoregressive Errors (Exact ML Method) J ≤ 2
AR (J)	Autoregressive Errors (Cochrane-Orcutt Iterative Method) J ≤ 12
AR (J)	Autoregressive Errors (Gauss Newton Iterative Method)
IV/ARJ	IV with AR Errors (Gauss Newton Iterative Method)
MA (J)	Moving Average Errors (Exact M2 Method) J ≤ 12
IV/MA (J)	With MA Errors; J ≤ 12

and produces a comprehensive set of diagnostic statistics and graphical analysis of these. Estimations can also be examined using the recursive and rolling regression procedures. The univariate procedures are excellent, well documented and the user interface very well designed. The Johansen routines in the package are only for the simplest of applications and in my opinion are probably best avoided as they may lead to wrong conclusions. Their analyses could of course be extended by saving error correction terms setting up single equations and testing some hypotheses using the single equation methods available. A new version of MICROFIT is promised for later this year and I understand that this has been greatly extended and improved.

PCGIVE[16] (and PCFIML) is based on the LSE econometric philosophy and we have given a brief description of its diagnostic tests for a single equation model. PCFIML is the equivalent for system estimation and is as comprehensive as regards methods and diagnostic tests and graphical analysis. It provides a comprehensive implementation of the Johansen methodology. This methodology is very involved and implementation problems are more likely to arise from a misunderstanding of the methodology than from the software.

REGX is a package written by Stephen Hall and could be described as an ideal package to accompany Cuthbertson Hall and Taylor (1992). Its general philosophy is again based on the LSE philosophy. However, in addition to the more usual single equation methods it provides time varying parameters, GARCH-M, seasonal estimation, general Kalman filter routines and matrix manipulation for extensions. While the interface is not as polished as MICROFIT or GIVE it provides, to academic users, some very useful routines at a very small cost.

RATS and SHAZAM are powerful flexible tools for statistical analysis. They are in effect programming languages with a large number of built in procedures. They operate by issuing commands at a command prompt rather than choosing items from a menu. Commands can also be gathered together in an external file which may also include control expressions (e.g. if ...then...else, do, block, and procedure definition statements). In theory, either of these packages could tackle nearly all the problems that one could encounter in econometrics. However, in practice, some problems may be too big or may require so much programming that they might not be suitable. RATS is particularly strong in time series analysis while SHAZAM is strong in diagnostic tests and rivals PC-GIVE in this regard. Both packages are used in the Bank and I have even used both together in a single job. The benefit obtained from learning both packages far outweighs the costs of trying to work with individual packages. RATS is widely used in the Bank. It has some very good time series data management routines and these are widely used in the Bank

Preliminary versions of CATS in RATS developed by Hansen and Juselius have been in circulation for some time. This package is a complete implementation of the Johansen procedure in RATS and a final version will be available about the same time as the promised new version of RATS[17].

The relative merits of command and menu driven systems are often debated. I confess to a certain preference for the flexibility of command driven systems but the faster learning curve means that where possible I would recommend a menu-driven system to a new user.

GAUSS is a complete mathematical programming language with a syntax that is a cross between FORTRAN, PASCAL and C. The basic element used in GAUSS is a matrix. Thus, even the most elaborate formulae can be written in one program line. GAUSSX is an econometric front end for GAUSS which provides many of the features of a more standard econometric package but with the programming facilities of GAUSS added. Among the procedures implemented in GAUSS in the Bank were Chow-Lin interpolation routines and an early version of the Johansen procedure. Gauss has wider usage than quantitative economic analysis and has been used in the for numerical analysis and optimisation

MATHEMATICA is a symbolic programming language. It is easier to explain this statement by way of an example

```
In[1] :=
        D[x^n,x]
Out[1]
          -1+n
        nx
In[2] :=
        Solve[a x^2 + b x + c == 0 ,x]
Out[2] =
          -b-Sqrt[b²-4ac]      -b+ Sqrt[b²-4ac]
{{ x -> ---------------},{-----------------}
               2a                   2a
```

In[n] := and *Out[n] :=* are respectively input and output to the program. The first example finds the derivative of x^n and the second solves the general quadratic equation. The program has the capability of doing very complicated symbolic mathematics.

The program is used by researchers, engineers and analysts in applications which span all areas of science, technology and business where quantitative methods are used. Varian (1993) describes the use of MATHEMATICA in Economics and Finance which alongside standard and Bayesian statistical techniques includes:

Diffusion Processes and Ito Calculus

Option valuation for Black-Scholes and binomial models

Modelling and solution of co-operative games

Nash equilibria

General equilibrium models

Optimal growth model

Solving linear discrete line models

Determining long run dynamics of economic models

Optimal incentive mechanisms.

Many other packages are available (e.g. TROLL, SAS, SPSS, TSP, ET, MATHLAB to name but a few). They all have their strong points (e.g. TROLL for macro-modelling, SAS or SPSS for general statistical work...)

Dewald, Thursby and Anderson (1986) is, to my understanding, the only major study of the efficacy of empirical economic work. They reported on the results of a Journal of Money, Credit and Banking survey on replication in empirical analyses. 154 authors were asked to submit their data and programs as used in articles published or submitted to the Journal. Of the 62 authors whose articles had already been published only 42 responded (mean response time 217 days). Of these 22 supplied data, the remaining 20 had either confidential data (2), lost or destroyed their data (14) or data available but not sent (4). 26 authors had their articles accepted but not published. The mean response time was 125 days. All but 1 responded. 5 did not supply data -

confidential	1
Lost	2
Data available but not sent	2

65 requests were sent to authors whose articles were under review. Only 49 replied and of these 1 had lost or destroyed his data and another 1 replied that his data was readily available but not sent. Considering that one authors of the study was an editor of the Journal it is surprising that one in four of these authors did not respond.

Data are of no use unless they are accurately recorded and properly documented. The first 54 data sets submitted were examined and only 8 met the required criteria. 14 data sets were incomplete. The main problems with documentation were failures to identify sources and problems with the identification of individual series.

An attempt was made to replicate in full nine analyses. Only two attempts were fully successful but these still required some assistance from the authors (mainly minor problems arising from the use of different computer systems). A third was successful apart from one equation which could not be replicated. A fourth had problems with numerical stability. The computer program used in the fifth was so defective that some of its results were useless. When the computer program was corrected the results did not change the conclusions of the article. The sixth, seventh and eight articles could not be replicated because of program problems, data problems and possible erroneous descriptions of the procedures used. The ninth article was an application of a large econometric model and the analysis could not be replicated with the resources available.

The authors recommended

> *"On the basis of our findings, we recommend that journals require the submission of programs and data at the time empirical papers are submitted. The description of sources, data transformations and econometric estimators should be so exact that another researcher could replicate the study and .... obtain the same results.... Alternatives must be proposed for authors whose research is based upon proprietary, licensed, or confidential programs and data sets .... Authors should submit the version .... of proprietary programs (such as SAS, SPSS, RATS and TSP) as well as listings of the instructions executed by the program".*

This recommendation or some variation of it has to some extent been adopted by several journals. The need to prepare data and programs in a form in which they may be replicated by others including possible referees should ensure that they are more correct and the results more reliable and more robust.

As an experiment I decided to replicate part of Lovell and Sellover (1994). They ask if state of the art econometric software is capable of contributing to serious error. They use four PC econometric packages -

MICROFIT	ver 3.0
RATS	ver 4.0
SHAZAM	ver 7, and
MICRO TSP	ver 7.03

to analyse three data sets

Longley (1967)

Dufour Gaudry and Tran (1980) and

A Consumption function data set derived form the Citibase Databank.

The Longley data set is very ill-conditioned. In my 1987 paper referred to already I used this data set and found that all the PC programs examined were capable of handling the Longley data far better than the programs Longley used. Longley shows the type of problems and numerical instabilities that can arise even in simple OLS regressions.

The Dufour et al (1980) dataset is known to have two local maxima of the objective function when a standard Cochrane-Orcutt procedure is applied. The true maximum lie at about p = 0.93 while the local maximum is at p = 0.33. When full maximum likelihood is used there is a single maximum at about p = 0.32. The table below compares my results with those of Lovell and Sellover. Apart from the method = HILU in RATs we are close to agreement.

Initially I had considerable problems replicating the Lovell and Sellover results. When I obtained a copy of their data on the interest and compared it to my own I found two discrepancies. Comparison with the original Dufour et al (1980) data set revealed that we had one mistake in each of our datasets. Using Cochrane-Orcutt in MICROFIT or the RALS procedure in PC-GIVE and an appropriate starting value I was able to get the true optimum. In the second and third column of results I give my results using the Lovell and Sellover data while the final two columns give the Dufour et al data.

There are some annoying, if not particularly serious, problems about these results. They can all be explained by realising that the different programs are maximising different objective functions and are using different criteria to assess convergence. I have managed to reconcile these in this case but one might have considerably more trouble in the more complicated problems likely to arise in practice.

Defour et al. (1980) Data Set-Autocorrelation Problem

		L&S Data		Revised Data	
	Lovell & Sellover	**Default Initial Values**	**Alter-native Initial Values**	**Default Initial Values**	**Alter-native Initial Values**
Exact AR(1)					
Inverse Interpolation	0.31661	0.31661		0.31664	.
Cochrane-Orcutt AR (1)		0.32883	0.92683	0.32889	0.9263
Maximum Likelihood	0.32921	0.32921	0.32921	0.32928	0.32921
Rats					
AR1 (METHOD = CORC)	0.3289	0.3289		0.3289	
AR1 (METHOD = HILU)	0.3289	0.9319		0.9318	
AR1 (METHOD = MAXL)	0.3166	0.3166		0.3166	
AR1 (METHOD = SEARCH)	0.3166	0.3166		0.3166	
SHAZAM					
AUTO	0.33629	0.33629		0.33632	
AUTO .../ML	0.31649	0.31649	0.3166	0.31653	0.31664
AUTO .../DROP	0.32867	0.32867	0.32888	0.32874	0.32895
AUTO .../GS	0.34	0.34		0.34	
AUTO .../ML GS	0.32	0.32		0.32	
AUTO .../GS DROP	0.93	0.93		0.93	
PCGIVE					
RALS		0.32888	0.93187	0.32895	0.9318

Journals publishing applied work should pay particular attention to these points. In general data and sources should be fully described. Estimation methods should be described in full and to an extent that it should be possible to replicate the analysis. Ideally data and programs should be supplied to the journal and available to a referee if required. In some cases editors might ask a referee to review the analysis. Only in this way can we be sure that empirical work is of the highest standard.

7. CONCLUSIONS

Econometrics or, more precisely, the application of statistics to quantitative economic analysis, is now in a state of flux. For a statistician it is probably one of the most interesting branches of statistics. In the last ten or so years there have been considerable developments both in theory and practice. In effect, theses developments are coming at such a rate that it is very hard to keep up with them. I do not expect that all of you will return to your computers in the morning and launch yourselves into the more advanced topics. I will be satisfied if I have increased your awareness of what modern econometrics is trying to achieve and that those who use it are more aware of the pitfalls involved. I would be pleased if this paper helped those who are responsible for the updating of the econometrics courses for students of Economics.

Perhaps, I might close by making a few comments on truth in science. The economist often looks to physics as an example of how theory should be developed and empirical work conducted. I am often very surprised that so many economists believe in the absolute truth of various 'laws' of physics. My own studies of theoretical physics in the sixties has left me with a healthy disrespect of such positivist views. Newtonian mechanics and the Newtonian theory of gravitation is an obvious case. Its success in providing solutions to many problems does not need to be elaborated here. In many cases its predictions are so close to reality that predictions and reality can not be distinguished. However, it is not reality and it has been known for more than a hundred years that it gives the wrong answer in certain cases.

Einstein (1905, 1916) proposed his theory of relativity as a solution to these problems. This theory encompassed the earlier Newtonian theory in that it explained all that Newtonian theory explained as well as various phenomena that the latter could not explain. However, we now know that relativity does not explain everything. It also is not absolute truth.

Thus, both Newtonian and Relativistic theory are not absolutely true. All of science and, in particular, physics advances by a process of approximation. Current theory is found to be defective and research leads to improvements. Bit by bit we advance towards the truth.

Economic theory is no different from other sciences. To-days theory may explain well certain facets of the economy and may provide useful forecasts and valuable insights. At first econometrics can demonstrate in what circumstances the theory is likely to be valid. Secondly econometrics provides the parameter estimates necessary to turn the theory into policy recommendations. Finally it can show where theories are defective and point the way to improvements. The profession has not reached agreement on how this can be done. In Section 5 we discussed the general to specific approach (Hendry), the fragility or sensitivity approach (Leamer), the macroeconomics and reality approach (Sims) and the macromodel approach. All four methods have added to our understanding. The user must choose whichever is best suited to has needs. If time were available he should try out more than one approach.

Finally there is the cautionary advice - possibly due to Leamer - in economics it is not too bad to sin if you know that you are sinning and preferably confess your sins on the spot. To sin without knowing that your are sinning is unforgivable.

Footnotes

1. Richard von Mises was an applied mathematician who first specialised in mechanics, hydrodynamics and the theory of flight. I understand that his book on the theory of flight from the early years of this century is still in print. As a practical man, in 1915, he constructed a 600 Horse Power airplane for the Austrian army and served in it as a pilot during the world war. His work on the foundations of probability began about 1919 and "Probability, Statistics and Truth" was originally published in 1928. In 1929 he published a large book on Probability and its application in statistics and theoretical physics. His "Mathematical Theory of Probability and Statistics" is based on lectures from the early 1950's. He identified himself with the positivist school of philosophy and has written on positivism. For a current assessment of his frequentist views on probability see von Plato (1994).

2. In recent times several laboratory style experiments have been carried out. Examples are the studies of the behaviour of trades conducted with undergraduate students in laboratories at Carnegie-Mellon University by Marimon and Sunder (1990) and at The California Institute of Technology by Aliprantis and Plott (1990). See also Chapter 29 of Azariadis (1993).

3. In practical work identification or lack of identification often leads to problems and errors in analyses. In the next Section of this paper there are some comments on identification in a cointegration context.

4. This is a simplification. The effect of exogenous variables may be included through the transfer function methodology.

5. The literature sometimes uses the term nonsense regression for what I call a spurious regression. The term spurious regression is then used to describe a regression between two variables that are related to one another only through a third variable.

6. See also Yule (1926)

7. The RATS program for the simulations and regressions is reproduced in Frain (1993).

8. There is a general misunderstanding that inducing trended variables in a regression is the cause of spurious regressions. None of the non-stationary variables included in these analyses are trended and yet the regressions are spurious. Here the random walk component of the variables is the source of the problem. Regressions between variables which are trended (in a deterministic

sense) may indeed produce nonsense regressions but standard statistical theory will apply. The problem may lie with the interpretation of the regression.

9. There may of course be other more basic problems arising from specification errors.

10. An argument against unit roots, which is frequently quoted, is that real interest rates are now the same as they were in Babylonian days. If real interest rates followed a unit root process the probability is very small that they are now at the same level as they were in Babylonian days. Therefore they could not follow a unit root process. This argument is wrong. Consider the Babylonian statistician who believed in unit roots. He would have predicted that real interest rates would be the same to-day as they were in his day. Yet we wish to discredit his theory on the basis that their predictions were true.

11. This model is of course too simplified to model reality but serves to illustrate some ideas.

12. Corresponds to $\widehat{\beta}$ in step 1 of the estimation procedure

13. A considerable amount of conditions are assumed and not elaborated. Further details are available in Priestly (1987)

14. Another approach to quantitative macroeconomic analysis is that described in Kydland and Prescott (1991). They choose the parameters of general equilibrium models of business cycles by calibrating the model to fit certain features of the business cycle.

15. There will always be a place for more powerful computers for work such as the type of multivariate time series analyses being completed at the Minnesota Federal Reserve Bank on a CRAY supercomputer.

16. PcGive (ver 9) runs under a new WINDOWS interface GiveWin. The full package includes OX an object orientated matrix programming language with a syntax similar to C++. A beta test version of PcFiml ver 9 has just been made available.

17. Since writing this RATS 4.20 for DOS and WINDOWS and CATS IN RATS have become available.

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