<u>Bond Portfolio Management</u>

Interest Rate Risk

Sensitivity

- *Inverse Relationship between Prices and Yields:*
  If the Price has fallen it implies the yield (over the remainder of the bond's lifetime) has risen (capital appreciation).
- *The price of a bond is more responsive to a fall in the yield than to a rise.*
- *The longer the life-time of a bond the greater the sensitivity to a change in interest rates.*
- *This sensitivity is increasing over the lifetime of a bond, but at a decreasing rate.*
- *Interest rate risk is inversely related to the size of the coupon rate.*

- *The higher the current yield to maturity, the less sensitive the price is to changes in the yield.*

<u>Duration</u>

Duration is a measure of the (weighted) average of time one has to wait to receive coupon payments.

The weight of each unit of time is given by the relative contribution of each payment to total payments (in present value terms).

Assuming continuously discounted yield to maturity:

$$P = \sum_{i=1}^{n} C_i \exp(-yt_i) = \sum_{i=1}^{n} PV_i$$

Differentiating this with respect to y:

$$\frac{dP}{dy} = -\sum_{i=1}^{n} t_i \left[C_i \exp(-yt_i) \right]$$

A change in P with respect to a (proportional infinitesimal) change in y is equal to the negative of a weighted sum of the present values of each payment.

Multiplying with *dy* and dividing by *P*

$$\frac{dP}{P} = -\sum_{i=1}^{n} t_i \left[\frac{C_i \exp(-yt_i)}{P} \right] dy = -\sum_{i=1}^{n} t_i \left[\frac{PV_i}{P} \right] dy$$

and defining Duration as:

$$D = -\sum_{i=1}^{n} t_i \left[\frac{PV_i}{P} \right]$$

We have that a relative change in *P* is a function of a proportional change in *y* and the duration of the bond.

<u>Macaulay's duration:</u>

A weighted average of the <u>times</u> of each payment made by the bond.

The weights are chosen to reflect the proportion of total payments that occur at that time. Here, we use the ratio of the present value of each payment to the price of the bond.

$$w_t = \frac{CF_t / (1+y)^t}{P}$$

Time we simply denote in numbers of periods from a certain point in time.

If a period is a year, and we are concerned with the *time* of the first payment of a semi-annual bond from now, then t = 0.5.

Thus, duration is equal to:

$$D = \sum_{t=1}^{T} t \times w_t$$

N.B.: $w_T > w_t$ for $t \neq T$.

Since the final payment of a bond's income stream includes the par value as well as a coupon payment.

N.B.: $$\sum_{t=1}^{T} w_t = 1$$

Since it is the sum of the percentage of each payment's present value in the present value of all payments.

$$\frac{\Delta P}{P} = -D \times \left[\frac{\Delta(1+y)}{1+y} \right]$$

Modified Duration:

Denote $D^* = D/(1+y)$ and $\Delta y = \Delta(1+y)$

$$\frac{\Delta P}{P} = -D^* \times \Delta y$$

Rules of Duration

1. Zero Coupon bonds' duration is identical to its maturity.

2. For bonds of equal maturity the duration is greater for bonds with lower coupon rates. (The final payment takes a larger share in total income).

3. Holding the coupon rate constant, the duration of a bond *generally* increases in time to maturity. This is always so for bonds trading above or equal to par.

4. Cet. par., a bond's duration is higher when its yield to maturity is lower. (Later periods are more relevant to

the present value of the income stream relative to earlier ones)

5. The duration of a perpetuity is $(1+y)/y$

6. The duration of an annuity is:

$$\left(\frac{1+y}{1}\right)\left(\frac{T}{(1+y)^T-1}\right)$$

7. The duration of a coupon bond equals:

$$\left(\frac{1+y}{1}\right)\left(\frac{(1+y)+T(c-y)}{c[(1+y)^T-1]+y}\right)$$

8. For coupon bonds trading at par this simplifies to:

$$\left(\frac{1+y}{1}\right)\left(1-\frac{1}{(1+y)^T}\right)$$

Convexity

The duration approximation of a bond's price sensitivity to a change in yields is derived from the tangency condition at $\Delta y = 0$.

Thus, for non-linear relationships, we must consider the fact that the slope is not only negative, but decreasingly so, i.e. we have a convex function.

Thus,

$$\frac{\Delta P}{P} = -D^* \times \Delta y + \frac{1}{2} Convexity(\Delta y)^2 .$$

The curvature of the % change in Price – change in yield curve is given by the convexity of a bond.

- Always positive for *positive* convexity.
- For small Δy the effect is negligible.

Convexity is a desirable property:

- The price gain for an increase in yield will be greater than the decrease in price for an increase in yield.

- For volatile interest rate environments we will have higher expected returns.
- Due to this *attractive asymmetry*, (should) trade at premium.

Negative convexity case?

E.g. Callable bonds.

Fund Management Goals

- Set performance standard, such as emulating interest rates
- Harmonise Cash-flow of Liabilities vs. Assets
- Outperform markets by trading on mis-pricing due to (assumed) incorrect market expectations re. interest rates or default (assume have superior knowledge/strategy).
- Reinvestment of income streams

FIS Fund Limitations

- There exist a large number of FIS with different characteristics, not only in terms of indenture but also in terms of default risk.
- Categorisation by credit rating, sector, country of origin, indenture, …
- Not all bonds readily available in proportions of market value.

Passive Strategies

Essentially, passive strategies imply that prices are assumed to reflect fair value.

Indexing

- The idea behind indexing is to replicate the performance of a given bond index.
- Generally, there exist a large number of constituent bonds in a bond index (+5000 for large funds).

- These are frequently updated in terms of the actual bonds included (mature bonds drop-out, new issues included) and the proportions in which they are included => rebalancing.
- Proportions (usually) reflect proportions in total market value.
- It is difficult to replicate the exact composition of the index.

Cellular Strategy

Replication is achieved by dividing the universe of bonds constituent in the index into sub-categories, such as the term to maturity, issuer's industry or country of origin, risk class…

These subcategories can be referred to as cells.

One can calculate the percentage of total index value accruing to each subclass. By holding bonds from each cell in these proportions one can emulate the actual index.

Thus, the essential characteristics of the index are reflected in the fund, and so the fund's performance should be similar that of the index.

Immunisation

- We have seen that bond values fluctuate with rate movements.
- Duration is important in assessing the exposure of the fund's value to fluctuations.
- The goal of immunisation is to isolate the value of a fund from interest rate movements.
- Depending on priorities, firms may wish immunise present (e.g. banks) or future (e.g. pension) funds values from fluctuations

Mismatch of cash-flows of liabilities and assets:
Banks tend to have short-term liabilities and long-term assets.
The differing duration of these implies imbalances in values are likely to occur if interest rates change.
Immunisation implies that imbalances are avoided.

Net worth Immunisation

Note that two types of risk are present:

Price versus reinvestment risk

Consider the movement of each of these:

- Price moves in the opposite direction to interest rates, i.e. a rate rise implies a capital loss.
- Future value of reinvested funds moves in the same direction of interest rates, i.e. future values of reinvested coupons grow faster due to interest rate hikes.

The fact that these risks move in opposite directions due to the same occurrence implies that they (partly) offset each other.

If the duration of a portfolio is selected such that it matches the horizon of the investment then price and reinvestment risk cancel out.

Convexity implies that this relationship is not exact.

Rebalancing is required

<u>Example</u>

Company sells 5 year zero, with YTM of 8%.

Funding?

Buy six year 8% coupon bonds and reinvest.

Scenarios:

If interest rates remain at 8%?

If interest rates change to 7%? ~ 9%?

Surpluses for Δr?

Convexity of the coupon bond is greater than the convexity of the zero.

Duration matched at $r = 8\%$ ($\Delta y = 0$), not across spectrum.

<u>Rebalancing</u>

For changes in r & T we need to rebalance the portfolio, as D changes.

Consider ΔT:

Obligation:

7 year zero at 10%:

10,000 today -> 19,487 @ year 7

Assets:

3 year zero and perpetuity at 10% yields

Cash flow matching and Dedication

Why not cover liabilities by purchasing bonds that have a cash-flow identical to the obligation?

If a series of future obligations are thus to be met this is called a dedication strategy.

Places restrictions on the type of assets that can be bought.

Pensions funds have obligations for which no bond exists.

Active Strategies

- Interest rate forecasting

Defalt premia

Homer and Liebowitz bond swaps categorisation:

1) Substitution
2) Interemarket spread
3) Rate Anticipation
4) Pure yield pick-up

<u>Horizon Analysis</u>

Riding the yield curve

<u>Contingent Immunisation</u>

<u>Interest Rate swaps</u>

Create synthetic fixed (variable) stream of liabilities if actual liabilities are variable (fixed).